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Scaling relation of earthquake seismic data

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HIGHLIGHTS

- The seismic data presents power laws and allometric laws.
- For the first time the rock mass data has been used to construct a weighted earthquake network.
- Connectivities and link weights both follow power law distributions.

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ABSTRACT

We study the spatio-temporal characteristics of earthquakes, and find that power laws and allometric growth laws are both statistically significant in the seismic dataset. For further analyzing the complexity of earthquake sequence, an approach of weighted earthquake networks modeling is presented with using the seismic and rock mass datasets. The rock masses covering the entire region are used to divide the region into a lot of small areas. It is shown that the distributions of connectivities and link weights both follow power law decay forms. The discovery of allometric growth laws is important for studying the dynamics of massive earthquakes. The suggested earthquake network is helpful to study the interactions between rock masses and expand a research prototype for modeling seismicity on complex networks.

1. Introduction

Earthquake is one of the most important and common natural phenomenon that directly affects human life and property. Therefore, scientists have been studying the genesis of earthquakes, earthquake forecast, hazard evaluation and other dynamics of earthquakes for a long history [1,2]. It is widely believed that seismicity can be characterized by extremely rich phenomenology, and then some known empirical laws are summarized, for example, the Omori law [3] for the temporal pattern of aftershocks showing slow relaxation and the Gutenberg–Richter law [4] for describing the frequency of

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tremors of a given magnitude. In addition, physicists take earthquake dynamics as a scale invariant process and study the correlation between different shocks. Some studies claim that the spatial positions of earthquake epicenters can form fractal sets [5,6]. The fractal dimension D proposed by Aviles, for instance, is used to measure the irregularity of the fault trace in the selected band and may changes significantly from the short-length band to the long-length band [5]. Kagan analyzed the fractal distribution of epicenters and discovered that seismicity is controlled by scale-invariant statistical distributions [6]. Burridge and Knopoff proposed a model for describing the slow creeping of the continental plates along the fault lines as a stick–slip process [7]. Bak et al. believed that the earthquake phenomenon can be regarded as a Self-Organized Critical process for studying the evolution of earthquakes and the scaling relations [8].

Although the correlations among different shocks have been studied, the actual mechanism of the underlying dynamics of this complex phenomenon has not been possible yet [1]. In a real complex system, detailed information on properties of system elements may not always be available, especially the interaction or correlation among them [9]. In such a situation, the network description offers a useful tool to build the relationships of system elements and mine the interactions between them. Scientists widely believe that complex network theory plays an increasingly pivotal role in revealing the complicated dynamics of real systems [10–13]. The vertices and the links connecting them respectively represent elements and their interaction or correlation. Therefore, we are able to grasp the gross property of the complex system by analyzing the structure of the network and the dynamics on it [9].

Using the complex networks theory, scholars have studied the spatio-temporal complexity of seismic data and have discovered some important features [1,14–21]. The scientists highlighted a novel aspect of seismicity as a complex phenomenon and found that the network associated with earthquakes presents the scale-free and small-world characteristics. Abe and Suzuki proposed a typical method for constructing earthquake networks [15]. In their studies, a geographical region is divided into a lot of small and regular cubic cells that each cell is regarded as a vertex. Every earthquake with any values of magnitude occurs within a single cell. A link connecting two vertices is generated as long as two successive earthquakes occur in the corresponding two cells. Then, the relationships between different cells can be constructed according to the earthquake sequence. Therefore, earthquakes and cells are integrated into a complex system that not only realize the modeling of temporal and spatial relationships of earthquakes, but also imply the interaction strength between cells from space perspective. Scientists studied the dynamics of earthquake networks with the change of cell size and found that the statistical features of earthquake networks may be different for different cell sizes [1,9,15,20]. These studies did not consider the underground structures or geological information. We believe that it is significant to introduce more geological information for the modeling of earthquake networks, which is helpful to reveal the dynamics of tectonic movement and space–time laws of earthquakes.

In this paper, we further analyze the spatio-temporal complexity of seismic data considering the magnitude, frequency, location of earthquakes. On the basis of this, this paper regards the earthquake network as a weighted network with using rock masses for dividing a geographical region rather than the segmentation method of cubic cells. It is widely believed that earthquakes mainly stem from the complex geological formation movement. The circulating interaction between rock masses may lead to earthquakes. Therefore, studying the relation between seismic data and rock masses becomes crucially important.

This study contributes to the literature in the following ways: (1) We reveal the allometric scaling pattern of the earthquake seismic dataset that describes the relationship between the average maximum magnitude and frequency of regions. (2) We present a new earthquake network modeling approach considering the earthquake seismic dataset and the geology data of rock masses. (3) This paper provides a research prototype, which can be used for further research on the spatio-temporal complexity of earthquakes.

The remainder of this paper is organized as follows. In Section 2, we analyze the spatio-temporal complexity of seismic data from earthquake frequency, magnitude and distribution perspectives. Section 3 illustrates the principle of earthquake network modeling with taking rock masses as the basic element of space dividing. Section 4 concludes the paper.

2. The spatio-temporal analysis of seismic data

Whether the seismic data imply some spatial and temporal characteristics, which is an interesting and significant subject. Therefore, in this paper, we study the complexity of the seismic data of Sichuan seismic station network which covers Sichuan and Chongqing. The research area is of 568,000 square kilometers and contains 44,091 earthquakes from January 1, 1990 to December 31, 2008. Obviously, Sichuan is an earthquake-prone area that about 190 earthquakes occurred per month during this period. Fig. 1 shows the spatial distribution of earthquakes in the region. We can see that most of earthquakes are intensively distributed in the Longmenshan Fault Zone, and there are few earthquakes in the eastern part of this region. The reason for this may be that great faults mainly concentrate on the northern, western and central parts of Sichuan, such as the Longmenshan Fault Zone, the Xianshuihe Fault Zone, the Jinshajiang Fault Zone, the Daliangshan Fault Zone and so on.

We study the relationship between the earthquake frequency and earthquake magnitude as shown in Fig. 2. The results indicate that the magnitude frequency distribution follows the Gutenberg–Richter law [4]. Gutenberg and Richter found that the relation between magnitude and frequency satisfies $\lg N = a - bM$. According to Fig. 2, we can see that small earthquakes occur more frequently and strong earthquakes rarely happen. From January 1, 1990 to December 31, 2008, there are 16 earthquakes that their magnitudes are equal to or greater than 6.0 ML, but 44,075 earthquakes that their magnitudes are

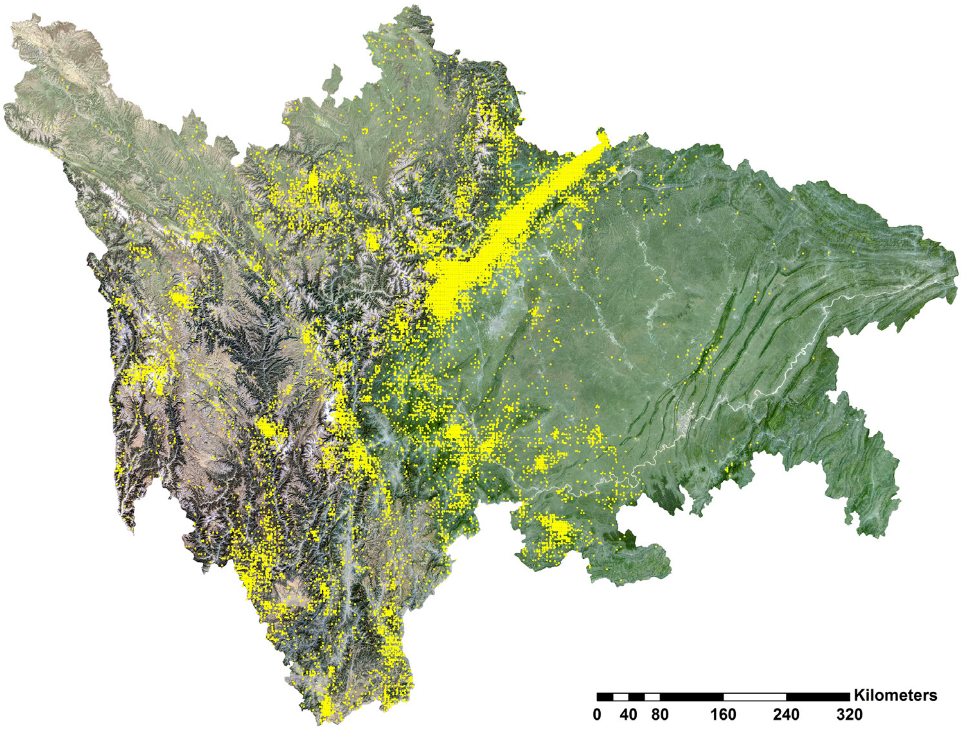

Fig. 1. Spatial distribution of earthquakes. The epicenters of earthquakes are indicated by the yellow dots. The seismic dataset is derived from the Sichuan seismic station network of the China Earthquake Networks Center (CENC) and is composed of 44,091 earthquakes' records, and the time period is from January 1, 1990 to December 31, 2008. The results show that most of earthquakes occurred in the Longmenshan Fault Zone.

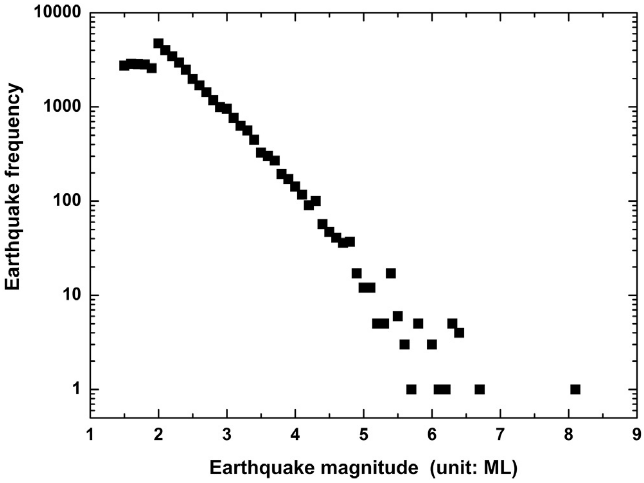

Fig. 2. Plot of the earthquake frequency versus the earthquake magnitude.

smaller than 6.0 ML. For this seismic data, the magnitude frequency distribution approximately follows $\lg N = 5.21 - 0.77M$. Although it is not practical to use this distribution function to predict earthquakes accurately, it is also helpful to reveal the complex dynamics of seismic data and the research of earthquake prediction.

Meanwhile, we calculate the number of sub-regions for different earthquake frequency where the sub-regions are the reference place of each earthquake occurring recorded by the Sichuan seismic station, such as Beichuan, Qingchuan, Pingwu and Mianzhu counties, etc. The statistical results as shown in Fig. 3 indicate that the relation between the number of sub-regions and earthquake frequency follows power-law distribution. Moreover, this distribution shows remarkable heavy tail phenomenon. Therefore, we can see that extreme events with a great deal of earthquakes actually occur more frequently

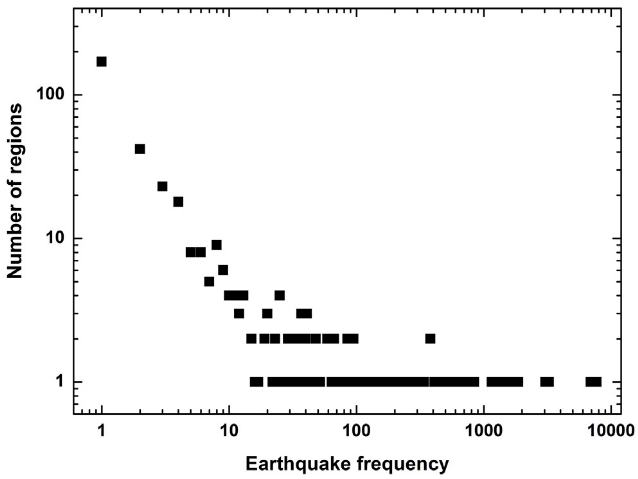

Fig. 3. Log–log plot of the number of regions versus the earthquake frequency.

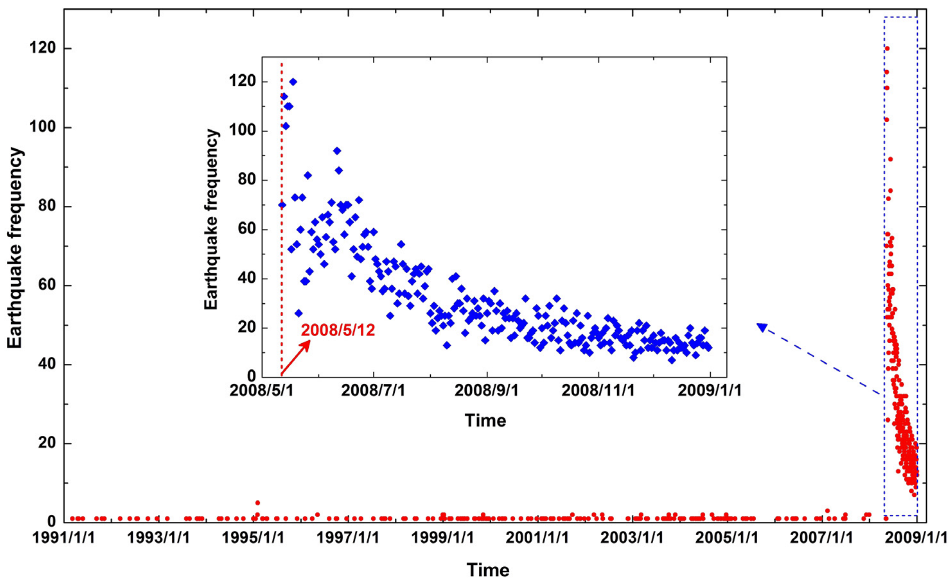

Fig. 4. Time series plots of earthquakes in Wenchuan County.

compared to e.g. in the normal distribution. We can get a very great value of earthquake frequency with a non-negligible probability. For this seismic dataset, there are 54 sub-regions (about 12.89%) that had occurred more than 100 earthquakes, the total number of earthquakes for the 54 sub-regions is 40,581. Consequently, earthquake events are frequent in the Sichuan Province, and significantly, most of earthquakes occurred in a small number of sub-regions.

Moreover, we further study the complexity of seismic data of Wenchuan County. Fig. 4 shows the time series plots of earthquakes, here the time period is from January 1, 1991 to December 31, 2008. The vertical scale represents the number of earthquakes per day. The statistical results indicate that the earthquake frequency is very low in a long period of time that just one or two earthquakes occurred each day. However, the earthquake frequency intensified sharply on May 12, 2008 that there have been 70 earthquakes in Wenchuan County. As we know that the "5.12" Wenchuan earthquake just occurred on this day. We believe that the crustal movement was very active at that time. A lot of aftershocks occurred in Wenchuan in the next few months. As time goes on, the earthquake frequency gradually decrease that means the strength of crustal movement is more and more weak. Therefore, we can conclude that before a strong earthquake happens, the earthquake frequency of this region is stable and very low; if a strong earthquake occurs, the frequency will mutate that tens or even hundreds of earthquakes occur every day.

One of the most important subjects in earthquake field is to study the earthquake frequency and magnitude of different regions. Here we count the earthquake frequency of each region having suffered earthquakes. The statistics shows that

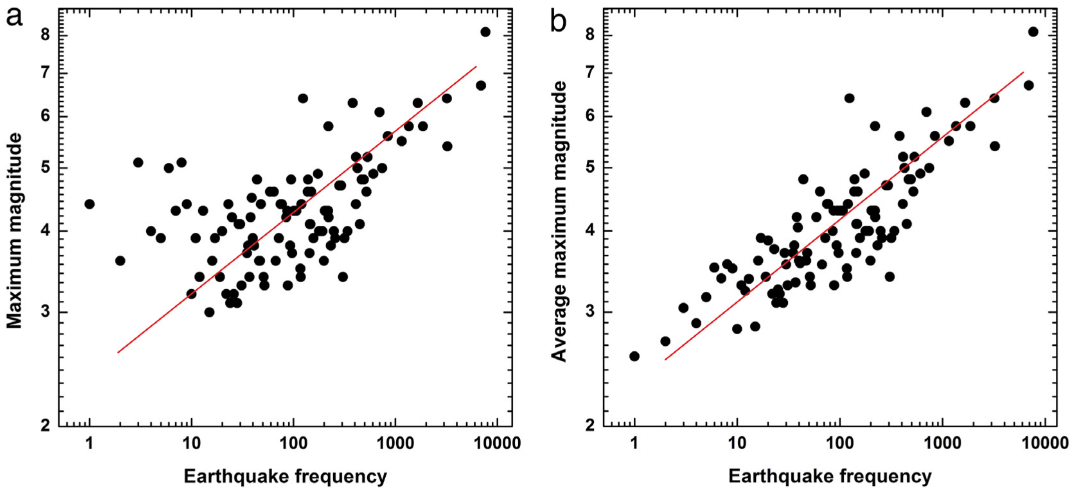

Fig. 5. Log–log plot of the maximum magnitude as a function of the earthquake frequency of regions.

7657 earthquakes occurred in Wenchuan where the frequency is greater than any other region in Sichuan. Qingchuan, Beichuan and Pingwu occurred 6923, 3237 and 3201 earthquakes, respectively. However, there may be several regions whose earthquake frequencies are the same but maximum magnitudes are different. Therefore, for a frequency with only one region, we just count the maximum magnitude; however, for a frequency with several regions, we both count the maximum magnitude and the average maximum magnitude. Therefore, the statistical results are shown in Fig. 5. Let M_{max} be the maximum magnitude of regions, $\overline{M_{max}}$ be the average maximum magnitude of regions, and ν be the earthquake frequency of regions. We can use $M_{max}(\nu) \sim \nu^{\alpha}$ to describe the relation between the maximum magnitude of regions and the earthquake frequency, and use $\overline{M_{max}}(\nu) \sim \nu^{\beta}$ to represent the relation between the average maximum magnitude and earthquake frequency of regions, where α and β are positive exponents. The statistical results indicate that both two relations obey allometric scaling pattern. It is obvious that α and β are the most important characteristics of the two relations. Surprisingly, we discover that α and λ are approximately equal, $\alpha \approx \beta = 0.13$. Therefore, we can approximately use $\overline{M_{max}}(\nu)$ to describe the relation between the maximum magnitude and earthquake frequency of regions, which can be defined as follows:

$$\overline{M_{max}}(\nu) \sim \nu^{0.13} \tag{1}$$

Allometric scaling is different from other scaling law phenomenon. Allometric scaling focuses on the power law relationship between two variables, however other scaling laws survey the power law distribution of single variable.

3. The weighted earthquake network

Some scientists have studied the network modeling of earthquake seismic data and revealed some interesting conclusions. Abe and Suzuki found that we can use the complex network theory to study the seismic data, and the earthquake networks satisfied the scale-free and small-world features [15]. These previous discoveries provided important theoretical support for further exploring the spatial–temporal characteristics and complexity of earthquakes. The modeling principle of the earthquake network in previous studies is as follows: The geographical region under consideration is divided into a lot of small cubic cells; each cell, in which events with any magnitudes occurred, is identified with a vertex; two successive events define an edge between two vertices, which means the complex fault–fault interaction is replaced by this edge; two vertices may sometimes coincide with each other (i.e., successive events occurring in the same cell), forming a loop. Through this subdivision method, different events are associated in time and space, and all the events constitute a dynamical network.

Although the previous studies revealed some important dynamics of earthquakes by grid partition strategy, we think it is necessary to consider the geological structure of regions, especially the distribution of rock masses. From the perspective of physics, earthquake is a kind of physical phenomena mainly triggered by the crustal movement. The interactions between rock masses can release enormous energy that may cause earthquakes. Therefore, earthquakes are related to rock masses to a great extent. Combined with the geographical information of rock masses and the seismic dataset, it is helpful to study the spatiotemporal characteristics of earthquakes and its relationship to rock masses. Therefore, this research has explicit physical meaning. On the basis of this, our proposal for constructing the weighted earthquake network is as follows. The geographical region is divided by different rock masses that these rock masses are adjacent to each other seamlessly and cover the whole geographical region; each rock mass, in which events with any magnitudes occurred, is identified with a vertex; an edge is generated between two vertices if two successive events occurred at these two vertices; the weight associated with a specific edge is defined as the total number of successive events between its two end vertices; if two

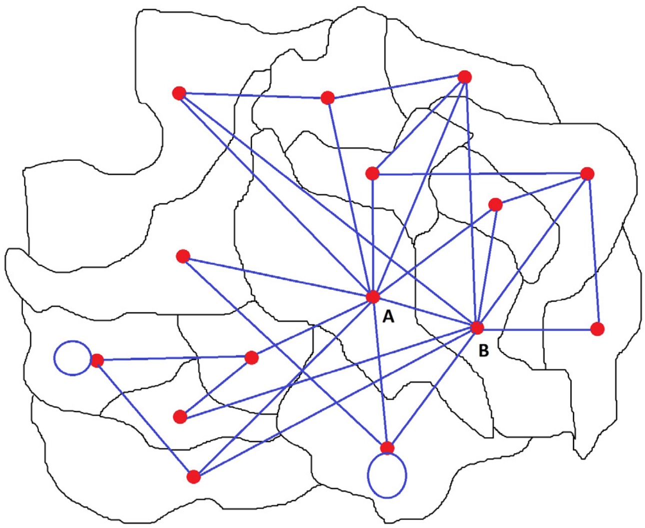

Fig. 6. The schematic description of the earthquake network. The polygons represent rock masses, the red vertices represent the earthquakes, and the blue links reflect the interactions between different rock masses and the spatiotemporal change of earthquakes. This figure shows that A and B are main shocks and have large degrees.

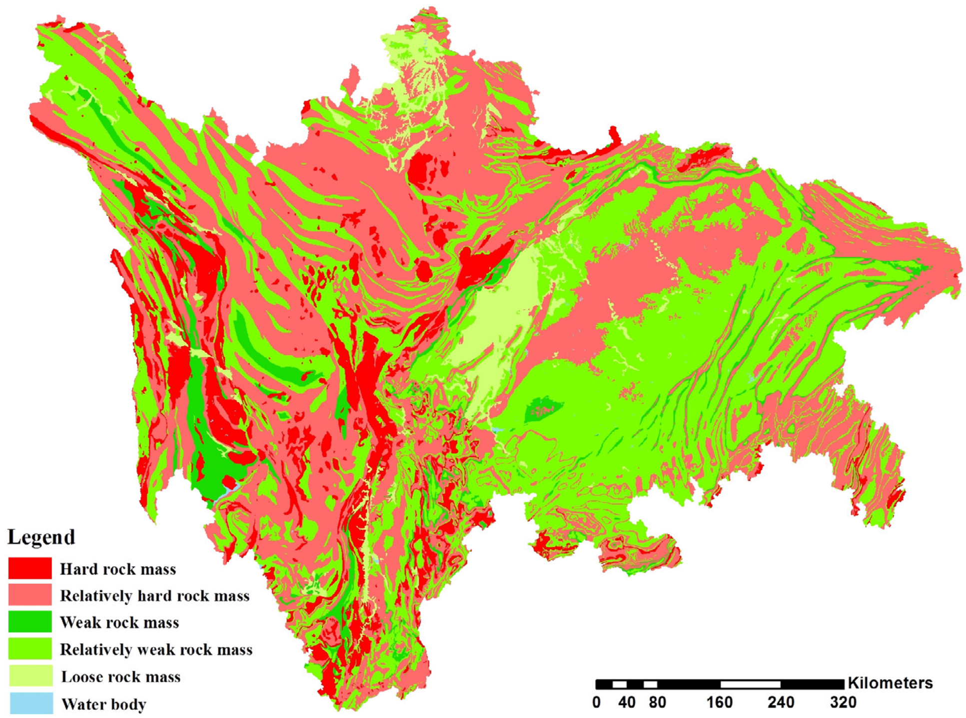

Fig. 7. Thematic map of rock mass distribution. This regional geological survey data covers the Sichuan province and Chongqing Municipality and is collected from the State Key Laboratory of Geohazard Prevention and Geoenvironment Protection of China. The data contains 6130 rock masses which are composed of 1109 hard rock masses, 2272 relatively hard rock masses, 259 weak rock masses, 2045 relatively weak rock masses, 241 loose rock masses and 204 water bodies.

successive events occurred in the same rock mass, then a loop (i.e., the two end vertices of this edge are the same vertex) should be generated. Fig. 6 presents the principle of earthquake network modeling, where a vertex corresponds to a unique rock mass and an edge connects two successive earthquakes.

The properties of rock masses must be obtained by geologists through regional geological survey, including the location, shape and lithology of rock masses, etc. Fig. 7 is the spatial distribution of rock masses covering Sichuan province and

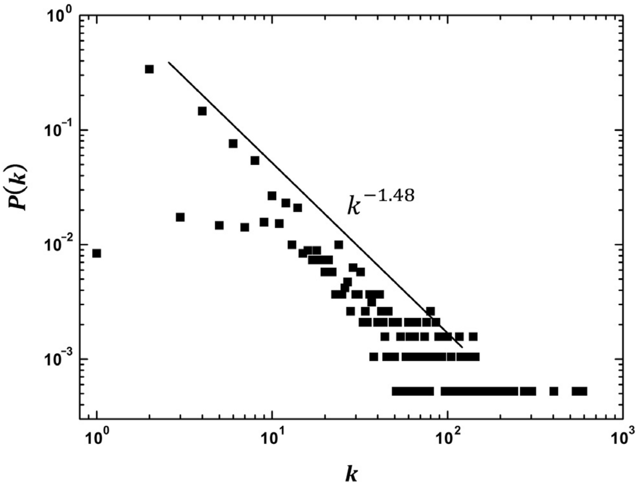

Fig. 8. Connectivity distribution of the earthquake network.

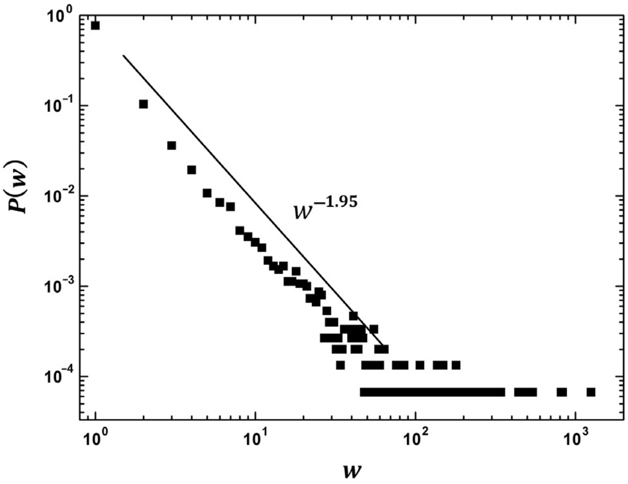

Fig. 9. Link weight distribution of the earthquake network.

Chongqing municipality. The results show that most of hard rock masses are distributed over the western, central and southern parts of this region, except for the eastern part. According to Figs. 1 and 7, we can see that the distribution of rock masses is consistent with that of earthquakes, which reveals some significant relationships between earthquakes and hard rock masses to a great extent.

On the basis of the seismic data and rock mass data of the study area, we construct the earthquake network consisting of 1913 vertices and 15,036 links, the average degree $\langle k \rangle = 15.72$. Fig. 8 shows the connectivity distribution of the earthquake network, which obeys power law distribution. The statistical result indicates that the earthquake network displays the scale-free nature and the power law exponent $\lambda = 1.48$. Therefore, the distribution function of the earthquake network follows $\mathrm{P}(k) \sim k^{-1.48}$. The previous studies have revealed the scale-free property of the earthquake seismic data, but the network modeling principle of seismic data is different from this paper and the power law exponents are significantly different. In the previous studies [1,13,14], the construction of earthquake networks is uncertain and depends on the cell size. For different cell size, the power law exponent is completely different and it may lead to uncertain conclusions. However, in this study, the earthquake network is constructed based on rock mass structure which can be obtained accurately by geologists through regional geological survey. Therefore, the earthquake network of this paper is definite. It is widely believed that the earthquakes mainly result from crustal movement. The proposed earthquake network is constructed based on rock mass dataset. Therefore, from the perspective of geoscience, our method is helpful to reveal the dynamics of earthquake events and tectonic movement.

In Fig. 9 we plotted the probability distribution $P(w)$ of link weights. The data appears to be described by a power law of the form $P(w) \propto w^{-\gamma}$. The slope of the distribution gives the estimation of the exponent $\gamma = 1.95$, therefore the distribution

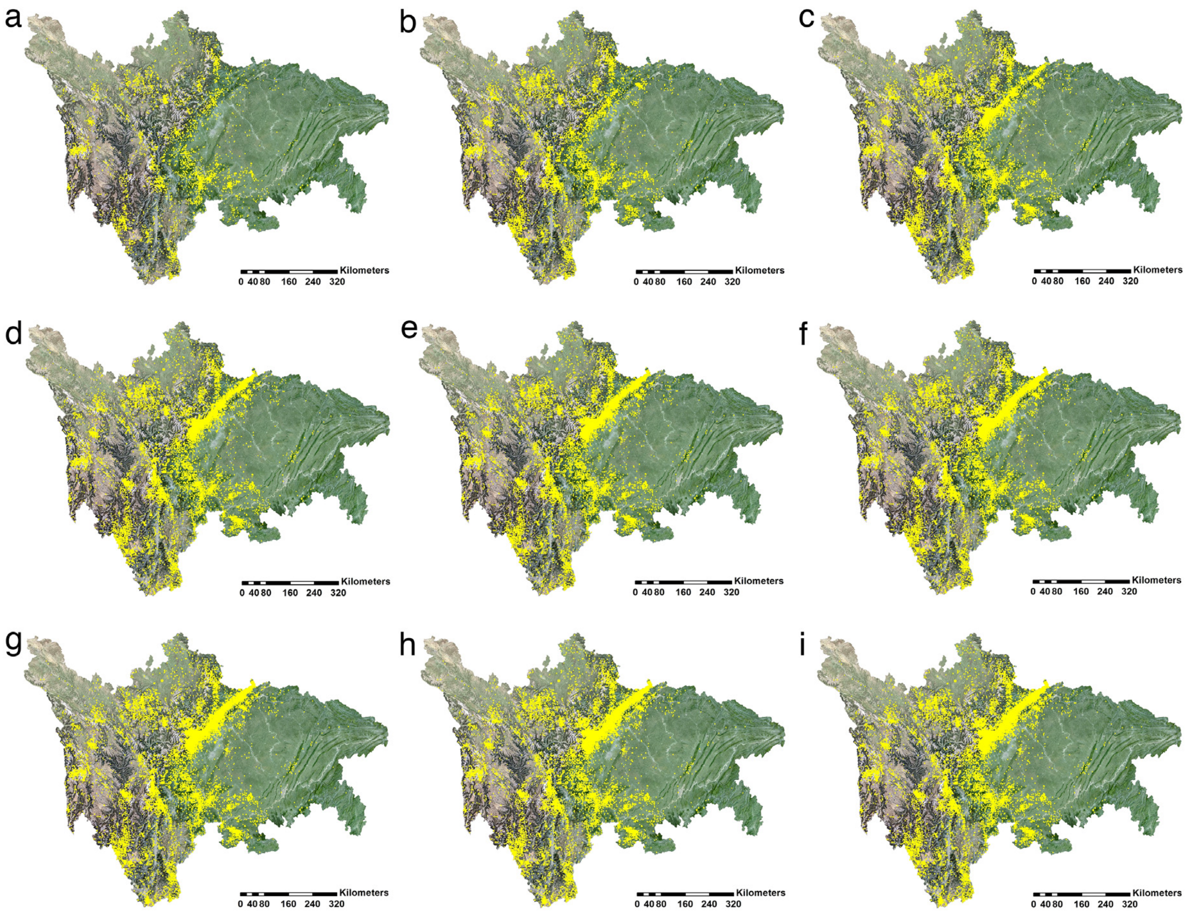

Fig. 10. Spatial distributions of earthquakes for different time periods. The starting time (January 1, 1990) is same for (a)–(i), but the ending time is different for (a)–(i). Here, (a)–(i) respectively contains 5000, 10,000, 15,000, 20,000, 25,000, 30,000, 35,000, 40,000, 44,091 earthquakes.

function can be defined as $P(w) \propto w^{-1.95}$. Link weight reflects the strength of the interaction between rock masses. Statistics show that 43 links whose weights are equal or greater than 100 and 11,585 links whose weights equal to 1. The results may indicate that there is no obvious relationship between most of rock masses, but there exists strong correlation between a few rock masses. More importantly, we found that the greatest weight is equivalent to 1252 and its two connected vertices are the same vertex, which means that there occurred 1252 successive earthquakes at the corresponding rock mass of this vertex. Therefore, we can conclude that the link weights of earthquake networks obey a power law distribution; strong correlation between vertices and self-correlation of vertices both exist in the earthquake networks, which is helpful to discover deeper dynamics of rock masses.

In order to further the characteristics of the earthquake network, this paper constructs several weighted earthquake networks for different time periods and calculates the statistical measures of each network, which can reflect the sensitivity of the earthquake network under spatiotemporal perturbation and help to mine the spatiotemporal features of seismic data. We construct 9 weighted earthquake networks respectively according to 5000, 10,000, 15,000, 20,000, 25,000, 30,000, 35,000, 40,000, 44,091 earthquakes with different ending time. The spatial distributions of these 9 groups of earthquakes are shown as in Fig. 10. Fig. 11 indicates the connectivity distributions of these 9 weighted earthquake networks. The results show that the patterns of these connectivity distributions are quite similar, although the networks have great differences in network size. The connectivity distributions all indicate power law properties and present heavy tail phenomenon. Although a lot of rock masses have small degrees, the rock masses with large degrees still cannot be ignored. Experiments show that it mostly exist interactions between rock masses with large degrees, because the weights between them are very great. This phenomenon indicates that the relation between active rock masses is tighter than others, and if an active rock mass occurs a quake, it will lead to the shaking of other active rock masses to a great extent.

Moreover, we compute some topological measure of these 9 weighted earthquake networks. Table 1 shows some statistical characteristics of above 9 weighted earthquake networks, where G (a)∼G (i) respectively represent the earthquake networks of (a)∼(i) in Fig. 10. Statistical results indicate that the average degree $\langle k \rangle$ has a biggest growth from G (a) to G (b). Besides, from G (d) to G (i), the average degree $\langle k \rangle$ and the average clustering coefficient $\langle C \rangle$ both approximately show a

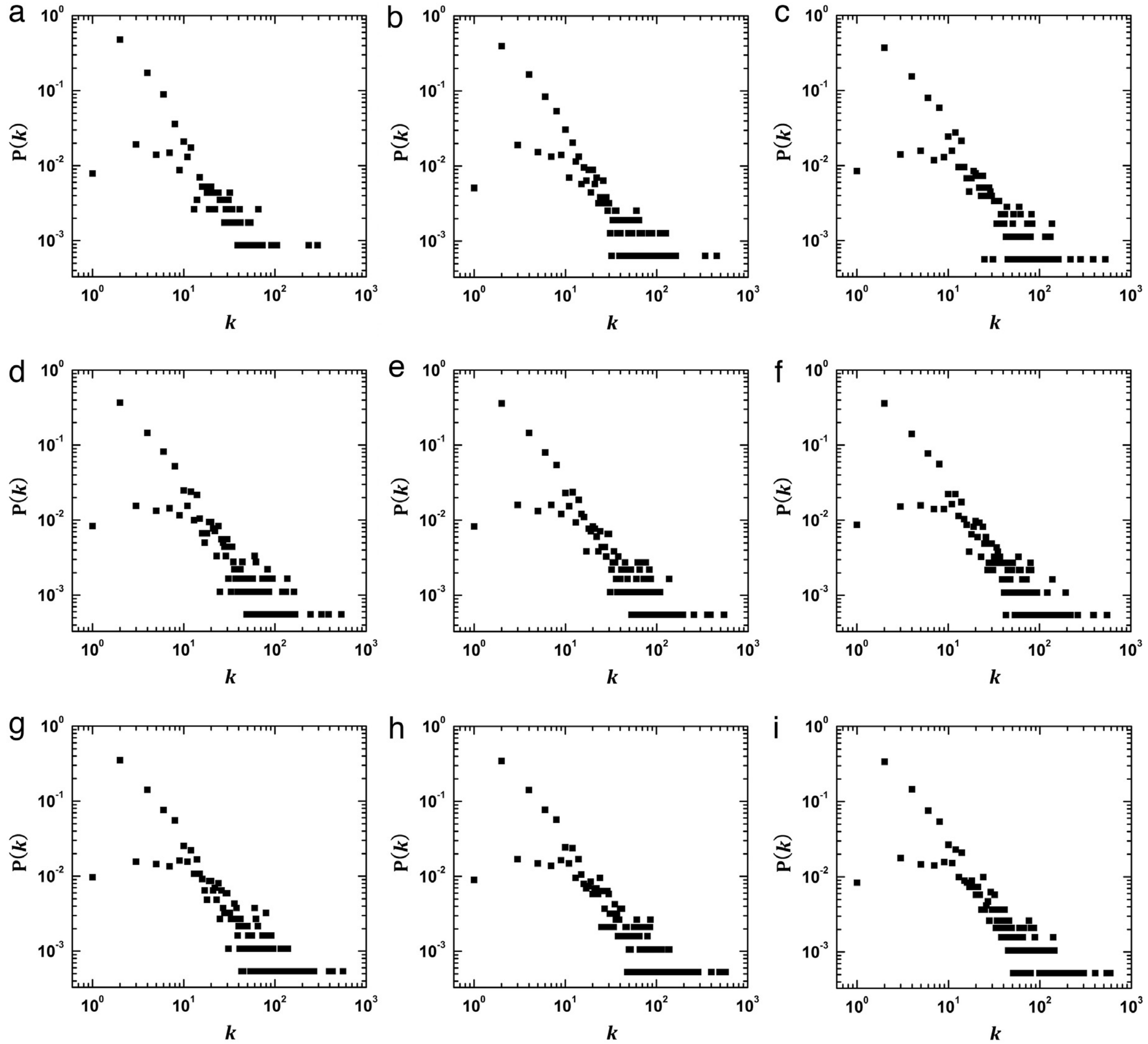

Fig. 11. Connectivity distributions of earthquake networks. (a)–(i) are respectively the connectivity distributions of earthquake networks constructed according to the 9 seismic datasets of Fig. 10.

Table 1
Statistical characteristics of 9 weighted earthquake networks.

Earthquake network	$\langle k \rangle$	$\langle C \rangle$	γ
G(a)	7.127	0.171	1.58
G(b)	10.065	0.192	1.55
G(c)	12.006	0.228	1.53
G(d)	12.813	0.241	1.49
G(e)	13.407	0.255	1.48
G(f)	14.048	0.268	1.48
G(g)	14.616	0.278	1.48
G(h)	15.234	0.296	1.48
G(i)	15.714	0.308	1.48

linear growth with the increase of the number of earthquakes, which means most of earthquakes occurred at the rock masses where had happened earthquakes in the past. Significantly, the power law exponents γ of these earthquake networks are approximately equal, especially for G (d) to G (i), which means the topological characteristics of these networks are very similar to each other. Therefore, we can conclude that although the number of earthquakes increases remarkably over time, the basic structural properties of the earthquake network approximately keep unchanged in this region.

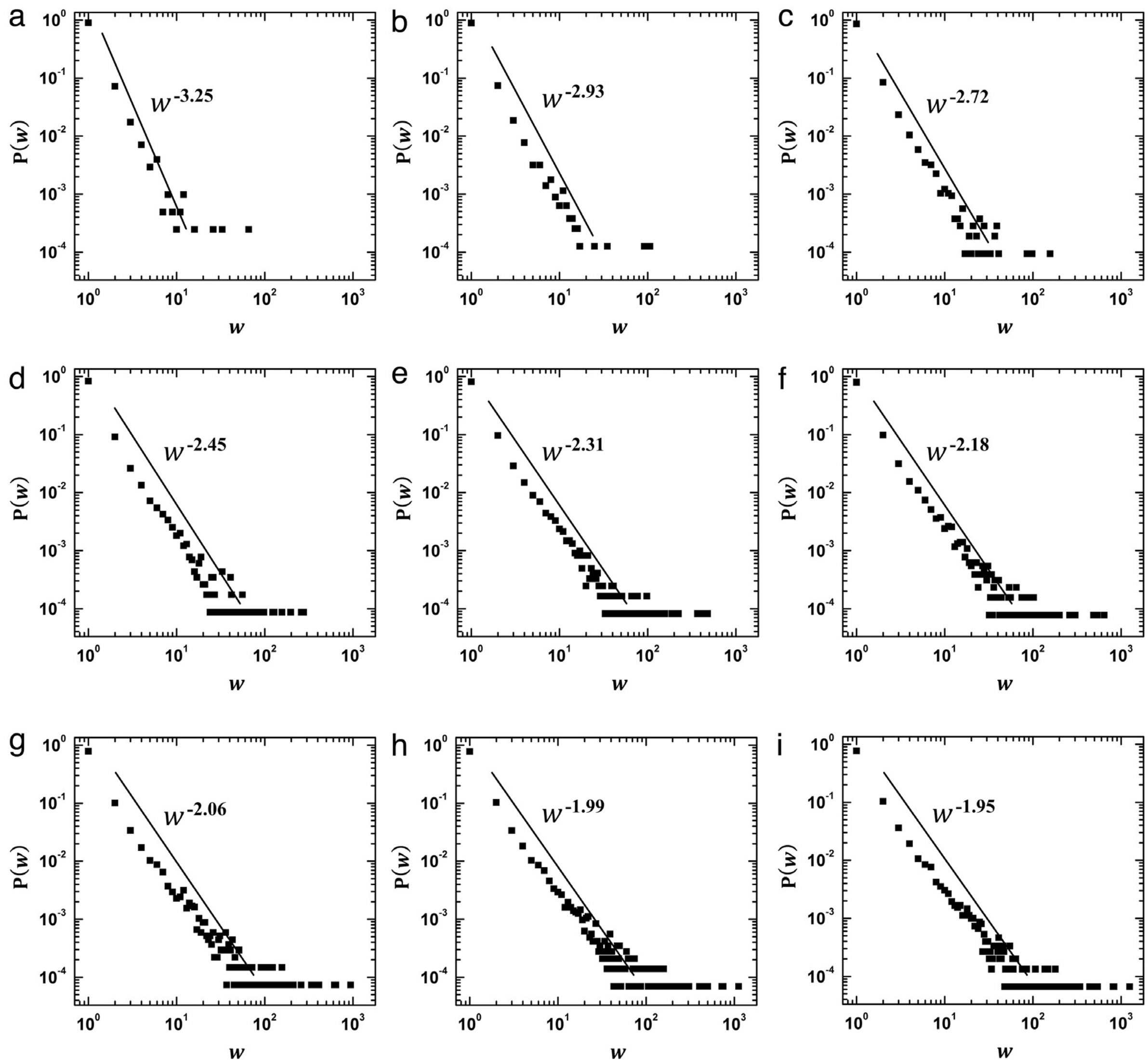

Fig. 12. Link weight distributions of earthquake networks. (a)–(i) are respectively the link weight distributions of earthquake networks constructed according to the 9 seismic datasets of Fig. 10.

Meanwhile, we study the weight distributions of above 9 earthquake networks as shown in Fig. 12. The results indicate that the weight distributions for different time periods all follow power law distribution and show heavy tail property. The maximum weight increases from 66 in Fig. 12(a) to 1252 in Fig. 12(i) that means the interactions between active rock masses are constantly undergoing over time. Besides, the power law exponent γ decreases from 3.25 to 1.95, and shows a stabilization trend during the decreasing process. Therefore, we can see that successive earthquakes are more likely to occur between the rock masses with large weight links. It can be concluded that if the link weight is very great, there will be more successive earthquakes occurring between the rock masses related to this link over time.

4. Conclusions

Studying the spatio-temporal dynamics of earthquakes is one of the most significant subjects in seismic research field. Complex network provides a useful technique to mine the spatio-temporal complexity of seismic data. In this paper, we analyze the complexity of seismic data and find that, it not only exists Gutenberg–Richter law such as the relationship between the earthquake frequency and earthquake magnitude, but also shows allometric growth laws for the relation between the average maximum magnitude of a region and its earthquake frequency.

For further studying the complexity of seismic data and the formation of earthquakes, we have presented an evolution model of earthquake networks based on complex networks theory with introducing geological data. This method constructs the relationships between earthquakes based on the seismic dataset and the geographic data of rock masses. According to the dynamical analysis of earthquake networks over time, we find out some significant evolution properties of earthquake networks, which is helpful to reveal the interactions between rock masses. The modeling idea is different from previous studies, because the entire region is divided by rock masses rather than a rectangular grid. The seismic dataset deriving from the Sichuan seismic station and the rock mass dataset providing by SKLGP (State Key Laboratory of Geohazard Prevention and Geoenvironment Protection) are used for experiments. The results indicate that the earthquake network can be described by a power law that the exponent is different from previous studies. Our work contributes to the spatio-temporal complexity research of earthquake seismic dataset and presents a new modeling idea of earthquake networks from the geological structure perspective.

Acknowledgments

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