Numerical modelling of fracturing effect stimulated by pulsating hydraulic fracturing in coal seam gas reservoir

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ARTICLE INFO

Article history:
Received 20 January 2017
Received in revised form
4 June 2017
Accepted 15 August 2017
Available online 7 September 2017

Keywords:
Pulsating hydraulic fracturing
Permeability enhancement
Fracturing mechanism
Numerical simulation
Fracturing parameter

ABSTRACT

Pulsating hydraulic fracturing (PHF) technology is an advanced permeability enhancement method for coal seam gas mining. Laboratory and field experiments indicate that PHF can stimulate a well-distributed fracture system inside a coal reservoir. However, the basic mechanism behind this effect is still poorly understood. In this study, a better mathematical model for pressure ripple propagation is proposed and an analytical solution is obtained. Furthermore, the particle flow code is applied based on the analytical solution to numerically simulate the fracturing effect of PHF. The mechanism for fracture system formation with the original coal cleat system is quantitatively analysed by using advanced indicators (crack event density, crack intensity rate and kinetic energy). A new cracking pattern is proposed and discussed. Eventually, fracturing effects under different engineering PHF inputs (i.e., pulsating frequency and ripple amplitude) are numerically simulated and analysed. The conclusions build a theoretical basis for the mechanism of PHF effect. The PHF parameters may also be largely improved and optimized for the extension and formation of fracture networks in a coal seam gas reservoir.

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1. Introduction

Hydraulic fracturing (HF) is a major mining technology for tight formation reservoirs (Al Rbeawi and Tiab, 2013; Gandossi, 2013) and has also been applied for coal permeability enhancement (Karacan et al., 2011). Fracture networks stimulated by HF mainly contain two types of fractures: main fractures and branch fractures. As the passageways of gas flow and diffuse (Li and Xie, 2004), well-distributed fracture networks, especially for branch fractures, can highly increase the permeability of a coal seam gas reservoir. Due to the fragility of coal, however, fracturing effects of traditional hydraulic fracturing methods are highly compromised (Jiang and Xing, 2016). Under relatively stable and high fracturing pressures, which are commonly used in HF, branch fractures do not have enough time to be fully developed (Huang et al., 2011). Hence, it is imperative to develop an advanced fracturing technology to form well-distributed, fully developed branch fracture networks in coal reservoirs.

In the 1980s, a new concept of pulse loading was initially introduced in HF process to improve the fracturing effect in the oil industry. By using this pulse loading method (e.g. explosion of charge located in the well or operation of submerged powder gas-generator) more and longer cracks could be produced (Sher and Aleksandrova, 2002; Wang, 1987). In order to make the pulse loading more applicable, the controlled pulse fracturing (CPF) was proposed to achieve the gradual pulse increase in the well pressure. In field tests, multiple radial vertical fractures are created by implementing CPF in wellbores, which effectively expand the fracture system (Uhri and Prairie, 1988). Lately, pulsating hydraulic fracturing (PHF) technology has been developed as an effective method to enhance permeability in a weak formation reservoir, especially for coal seam gas reservoirs. The fundamental idea of PHF is to create fluid pressure ripples by injecting fracturing fluid with a certain pulsating frequency. Under this low periodical pressure loading, a well-distributed fracture network can be produced with fully extended branch fractures and the rupturing progress of coal mass can be accelerated. A few laboratory studies have been conducted to seek the PHF mechanism. A variable frequency pulse hydraulic fracturing testing system was built (Li et al., 2013) and the influence of pulse frequencies for fracturing effects was studied (Li et al., 2014). The pulsating water pressure propagation in coal fractures during PHF was experimentally studied by using a simplified model (Zhai et al., 2015).
Due to the complexity of field conditions (geological structure, crustal stresses etc.), attempts to approximate the field PHF effects in laboratory studies are restricted. The basic mechanisms for pressure ripple propagation and formation of a fracture network are still poorly investigated. Particularly, the impacts of the original coal cleat system for fracturing effects have not been considered in experiments because of the existing experimental limitations. Therefore, numerical simulation is introduced as an alternative and effective way to study the basic mechanism of the PHF fracturing effect. Current numerical schemes for HF cannot be directly used in PHF simulation due to the following two reasons: (1) common numerical methods (e.g. FEM and FDM) for HF simulation (Guo et al., 2015; Zhang and Bian, 2015; Zhao et al., 2014; Zhou and How, 2013) are not suitable (at least not directly) for the simulation of PHF. The details are further discussed in the later part of this paper; (2) pressure ripples after propagation in the fracturing pipe are different from the input pressures. An accurate analytical solution for pressure ripples is required to approximate the actual loading condition of PHF.

To solve these problems proposed above, an advanced mathematical solution for the propagation of pressure ripples is proposed. Based on the analytical solution, numerical simulation is conducted by using the three-dimensional particle flow code, which could well characterize the formation, development, and connection of micro-cracks, and the stress distribution affected by the interaction of fractures. The original cleat system and the contacting relationships are fully considered in the coal model; tri-axial loading is introduced to simulate crustal stresses. Fracturing effects under different input schemes and parameters (i.e. pulsating frequency and pressure amplitude) are simulated for engineering optimization. The simulation results are expected to provide a valid explanation for the fracturing mechanism of PHF.

2. Pulsating pressure and propagation

PHF technology uses a piston pump to create sinusoidal water pressure at the entrance of a fracturing pipe. This pressure change can propagate inside the pipe and generate pressure ripples (Bergada et al., 2012; Harrison and Edge, 2000). Compared with HF, the sinusoidal pressure has relatively lower value with more drastic pressure changes (Li et al., 2014; Zhai et al., 2015). Unlike the traditional fracturing method, which relies on seepage to propagate pressure, the PHF allows fracturing pressures to propagate in a form of mechanical wave with a velocity that much higher than the velocity of seepage. Hence the changing pattern of pulsating pressure ripples at the end of a fracturing pipe has a major impact on fracturing effects. It is imperative to give the valid mathematical solution for engineering optimization. Many mathematical models have been proposed to describe water pressure changes in narrow fractures. The percolation model (Rehbinder, 1977) and transient flow model (Fiorotto and Rinaldo, 1992) are the most acknowledged models. These models are effective approaches for pressure propagation in dam fractures and long water pipes, both considering the velocity of water flow inside the fracture pressure (fracturing pipe is unsaturated and has relatively large physical scales); the high-frequency vibrations that are generated from inherent frequencies can also be numerically calculated from these partial differential equations (PDE). Regarding PHF, however, these models are not appropriate for describing the patterns of pressure propagation. Under PHF stimulation, a fracturing pipe is filled with fracturing fluid within a short pulsating time and the fracturing fluid is confined in an extremely narrow space with high pressure. Meanwhile, PHF uses low frequencies (e.g., 15 Hz, 20 Hz, 25 Hz, etc.), which make the wavelengths of pressure ripples $10^2$ to $10^3$ times larger than the geometrical scale of coal fractures. Hence, the fracturing fluid flows for pressure propagation are negligible and should be eliminated from the mathematical model.

### 2.1. Analytical solutions for pressure propagation

A straight pipe filled with water should be simplified as a one-dimensional mode. By solving the elastic wave equation in one dimension, the analytical solution for a water-filled pipe’s longitudinal vibration can be accurately obtained. A uniform mode is considered to represent the longitudinal vibration of a straight water-filled pipe. The governing PDE can be written as

$$\rho_w \frac{\partial^2 u}{\partial t^2} - \lambda \frac{\partial^2 u}{\partial x^2} = 0, \ 0 < x < L, \quad u(x,0) = f(t), \quad u_t(x,0) = 0.$$  

(1)

where $u(x,t)$ is the longitudinal displacement of the water pipe at distance x and time t, $\rho_w$, $\lambda$, and L are the density, bulk modulus of fluid and length of pipe, respectively. The boundary conditions and initial conditions can be written as

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = f(t) = \frac{1}{\lambda} \left( B_0 + A_0 \sin(\omega t) \right), \quad u(L,t) = 0.$$  

(2)

Physically, boundary conditions represent a free-clamped water pipe. The fluid entrance is set as a free boundary for pressure input, with coefficient $A_0$, constant parameter $B_0$ and pulsating angular frequency $\omega$. The end of a fracturing pipe is set as a zero displacement boundary because the PDE discusses the pressure propagation prior to fracture extension. The solution for equation (1) is assumed as

$$\frac{\partial u}{\partial t} = U_1(x,t) + U_2(x,t).$$  

(3)

where $U_1(x,t)$ is the general solution and $U_2(x,t)$ represents a particular solution. The general solution can be obtained by using the method of separation of variables, and after reducing the boundary conditions to homogeneous conditions, the general solution can be written as

$$\frac{\partial^2 U_1}{\partial x^2} = \sum_{n=1}^{\infty} C_n \sin(\omega_n t) + D_n \cos(\omega_n t) \cos \left( \frac{\omega_n x}{L} \right),$$  

(4)

$$\omega_n = \frac{(2n-1)\pi}{2L}, \quad \omega_\beta = \frac{(2n-1)\pi}{2L}. \quad (5)$$

where $\omega$ denotes the velocity of pressure propagation in the water-filled pipe model; $C_n$ and $D_n$ are the coefficients for the solution.
$U_f(x, t)$, which are related to initial conditions calculated from the particular solution.

$$C_n = 2 \int_0^L \left[ \left( \frac{\partial U_f}{\partial t} \right) \cos \left( \frac{\omega_n x}{a} \right) \right] \, dx = \frac{16 \pi a \alpha L^2}{(2n - 1)^2}$$ \hspace{1cm} (6)

$$D_n = 2 \int_0^L \left( -U_f(x, 0) \cos \left( \frac{\omega_n x}{a} \right) \right) \, dx = \frac{8 \pi L^2}{(2n - 1)^2}$$ \hspace{1cm} (7)

Similarly, the particular solution in equation (9) can be written as

$$U_2(x, t) = B_0(x - L) + A_0 \sin(\omega t) \left( \frac{a}{\omega} \sin \left( \frac{\omega_n}{a} x \right) - \frac{a}{\omega} \tan \frac{\omega_n}{a} \cos \left( \frac{\omega_n}{a} x \right) \right).$$ \hspace{1cm} (8)

The pressure changes can be calculated by solving the displacement equation:

$$P(x, t) = \int \frac{du}{dx}$$ \hspace{1cm} (9)

2.2. Solution analysis

The fully saturated pipe model is proposed so that the high-frequency components of the pressure ripple in a fracturing pipe, which are observed from experiments and field tests (Li et al., 2014; Wang et al., 2014, 2015), can be accurately obtained from this solution. Fig. 1 shows the non-dimensionalized input pressure and ripples at the end of a fracturing pipe. The dashed line denotes the input sine function $f(t)$, with ripple amplitude $A_0$ and parameter $B_0$. The specific values of both parameters are decided through the designation of the piston pump of the pulsating machine. The solid line represents the analytical solution of pressure ripple propagation, using $f(t)$ as the pressure boundary condition. Mathematically, the high-frequency components are included in general solution $U_f$, while the coefficients $C_n$ and $D_n$ are determined by the general solution and geometrical parameters of the pipe model (i.e., equation (6) and equation (7)), respectively.

From a physical perspective, $\omega_n$ represents the inherent frequency of the water pipe, which means that the vibrating responses for pressure ripples can only possess specific frequencies. These parameters are determined by the physical properties and the length of the water-filled pipe. The integer $n$ denotes the scale of the inherent frequency; the frequency is higher with larger scales. Under continuous pulsating, pressure ripples propagate through the fluid and reflect back and forth in the pipe. Ripples cannot simply maintain their original sinusoidal shape but rather gradually form a standing wave inside the pipe. The frequency of this standing wave is assembled by a series of harmonic waves with different inherent frequencies and the input pressure. Hence, the high-frequency components are generated. In this research, the pressure ripple calculated from the analytical solution is used for numerical simulations.

2.3. Limitation of the solution

The boundary conditions are valid for these PHF laboratory experiments. Because they are conducted in a small coal sample that seal by steel plates to load the surrounding stress (Li et al., 2014, 2013); or in a sealed steel pipe (Zhai et al., 2015). In these cases, the wave energy is largely confined by strong impedance (e.g., steel), which can be approximated by a boundary condition of a zero displacement. By using the Zoeppritz equations, the proportion of wave energy is calculated. The equation can be written as

$$\begin{bmatrix}
\sin \alpha & \cos \beta & -\sin \alpha' & \cos \beta' \\
\cos \alpha & -\sin \beta & \cos \alpha' & \sin \beta' \\
\sin 2\alpha & \frac{Vp_1}{Vs_1} \cos 2\beta & \frac{Vp_2}{Vs_1} \sin 2\alpha - \frac{Vp_1}{Vs_1} \sin 2\beta \\
\cos 2\beta & -\frac{Vp_1}{Vs_1} \sin 2\alpha & \frac{Vp_2}{Vs_1} \cos 2\beta - \frac{Vp_1}{Vs_1} \cos 2\beta' \\
\end{bmatrix}
\times
\begin{bmatrix}
\frac{t_s}{t_{ps}} \\
\frac{t_s}{t_{pp}} \\
\frac{t_s}{t_{pp}} \\
\frac{t_s}{t_{ps}} \\
\end{bmatrix}
= \begin{bmatrix}
-\sin \alpha & \cos \alpha & \sin 2\alpha & -\cos 2\alpha \\
\end{bmatrix}
\hspace{1cm} (10)
$$

where $\alpha$ is the incident and reflected P wave angle ($t_{pp}$), $\beta$ is the reflected S wave angle ($t_{ps}$), $\alpha'$ is the refracted P wave angle ($t_{pp}$), and $\beta'$ is refracted S wave angle ($t_{ps}$). The subscript ‘P’ and ‘S’ denote the wave type; the ‘1’ and ‘2’ represent the different medium. The $t_{pp}$, $t_{ps}$, $t_{pp}$ and $t_{ps}$ is the rate of displacement amplitude versus incident pressure amplitude. For the PHF, the water pressure wave is P wave with a zero incident angle, so the all wave angles equal to zero. Fig. 2 exhibits the wave energy rate with five different wave impedance based on Equation (10).

It can be observed that nearly 90% of wave energy will be reflected back by water/steel impedance and 80% for the coal/steel impedance. Hence the boundary condition with a zero displacement is a valid approximation, and the solution proposed in this paper is good for the explanation and numerical simulation PHF laboratory results.

Meanwhile, it must be pointed out that water/coal impedance cannot be treated as zero displacement; because nearly 90% of the wave energy propagates into coal medium and only 10% of the wave energy can be reflected back. Therefore, in real coal reservoir, the high frequency cannot be naturally generated; so the fracturing effect may not be as good as that of the laboratory. Nearly all of the literature fail to take this energy dispersion effect into consideration.

However, since the latter part of this paper indicates that a better fracturing effect can be achieved by pressure wave with high frequency, a ‘seal-release’ pulsating scheme should be used to improve the PHF technology. The scheme means to seal the fracturing pipe with steel for a certain time to generate high-frequency components, and then release the new pressure wave form into coal reservoirs. By using this scheme the high-frequency components that same with laboratory experiment can be produced.

3. PHF simulation and mechanism interpretation

To properly understand the mechanism of the fracturing effect stimulated by pressure ripples in coal reservoirs (the “coal mass” could be replaced by other mediums by modifying the mechanical properties), the interactive relationships between coal properties, coal cleats, and fluid pressures need to be clearly discussed. However, existing experimental apparatuses cannot directly observe
these phenomena, which is relatively easy to achieve via numerical simulations. Discrete element methods (e.g. the particle flow code, PFC3D) possess unique advantages for simulations concerned with the formation, growth, and eventual interaction of micro-cracks (Mas Ivars, 2010). Meanwhile, they can define and build the particle models that fully consider the original coal cleat system. In this paper, advanced PFC3D model is proposed to research the fracturing mechanism by PHF stimulation. The proposed analytical solution, which includes the high frequency components of fracturing pressures, is adopted as input functions and studied within comparative situations. Mechanical behaviours (velocities, energy, and cracks) under pressure ripples are accurately measured. Hence, the basic fracturing mechanism for PHF can be properly observed and discussed.

As we discussed early, the finite-element-related methods could also be applied to simulation the PHF. However, there are major problems that make the FEM not as suitable (or practical) as DEM. First of all, the wavelengths are required to fulfill CFL (Courant-Friedrichs-Lewy) condition and grid dispersion condition to conduct a stable simulation, which means in order to simulate the fracturing effect of high-frequency components the number of temporal steps and grids in FEM needs to be dramatically increased. It cannot be achieved just by using mesh refinement because this numerical stability for wave propagation requires the whole model to fulfill the CFL condition. Secondly, the DFN model, which is crucial for the simulation of PHF in a coal sample, is not very easy to be

![Fig. 1. (a) Input pressure and pressure ripples at the end of the fracturing pipe, (b) enlarged circular area for the high-frequency components.](image-url)
achieved on the FEM; the highly fractured model easily cause difficulty for the convergence of stiffness matrix. Meanwhile, the mesh distortion is also hard to avoid with high-frequency components.

The basic mechanism of PHF is fundamentally different with the traditional HF. Therefore, for better calculation efficiency and numerical stability, the discrete element methods are recommended and adopted in this paper.

3.1 Implementation of hydraulic coupling in PFC3D

The hydraulic coupling algorithm is introduced in the program to simulate the hydraulic fracturing effect (ICG, 2008). A compacted, bonded assembly of particles is firstly generated. The void geometry in an assembly of four neighbouring particles is regarded as being identical to the actual space between particles. A “domain” is defined as a pore created by every four neighbouring particles (Fig. 3a) such that each link in the chain is a side of a tetrahedron (Li and Holt, 2002). Each link (termed flow channel) between two adjacent domains has a small space between three neighbouring balls (Fig. 3b). As far as fluid is concerned, the flow channel is equivalent to a cylindrical pipe with length $L_p$ and aperture $w$. The flow rate (volume per unit time) in a flow channel is given by

$$q = kw^3\frac{\Delta P}{L_p},$$

(11)

where $k$ is a conductivity factor; $\Delta P$ is the pressure difference between two adjacent domains. The length of the flow channel $L_p$ is considered to be the distance between the centres of the adjacent domains in question.

Each domain receives flows $\sum q$ from surrounding channels. In one time step, i.e., $\Delta t$, the increase in hydraulic pressure is given by the following equation, assuming that inflow is taken as positive:

$$\Delta P = \frac{K_f}{V_d} (\sum q \Delta t - \Delta V_d),$$

(12)

where $K_f$ is the fluid bulk modulus and $V_d$ is the apparent volume of the domain. The second term ($\Delta V_d$) represents the mechanical change in volume of the domain, i.e., the mechanical changes in domain volumes cause changes in domain pressures.

Each domain accumulates the hydraulic pressure $P$, which exerts tractions on the enclosing particles. The hydraulic pressure in a domain is uniform and the tractions are independent for the path around a domain. If the polygonal path joins the contacts surrounding a domain, the force vector on a typical particle is

$$\mathbf{F}_i = P\mathbf{n}_i s,$$

(13)

where $\mathbf{n}_i$ is the unit normal vector of the line joining the centres of a domain and a particle; $s$ is a projective area on the particle in a domain.

Fluid flows and fluid-mechanical coupling effects are calculated at each time step. Particles are subjected to the laws of motion; the relative motion of two adjacent particles causes overlap or isolation, which generates contact forces; local contact forces break bonds under the cracking criteria from Ma et al. (2016, 2015).

3.2 Establishment of PHF simulation

3.2.1 Coal model

The synthetic rock mass (SRM) model provides the ability to conduct numerical experiments by combining two simulation
approaches (Mas Ivars et al., 2008): the Bonded Particle Model (BPM) and the Discrete Fracture Network (DFN). The BPM uses the particle flow code to assemble and bond particles in three dimensions (Potyondy and Cundall, 2004). The DFN is inserted into the BPM by considering the distribution of smooth joint (SJ) contacts (Mas Ivars et al., 2011; Mas Ivars, 2010), and thus creating a synthetic coal mass. Fig. 4a shows the SRM coal model for PHF stimulation. More than 64,000 particles (maximum radius 6.5 mm; minimum radius 4.6 mm) are assembled into a cube with the length 500.0 mm; porosity of the coal model is approximately 12%. Parameters for the model’s micro-structures are listed in Table 1.

Three sets of coal cleat are incorporated in the model: bedding plane, face cleat and butt cleat, which are built inside with non-uniform spacing of 50.0 mm, 40.0 mm and 80.0 mm, respectively. The smooth joint contact defines the contact relationship in coal cleats and correlative parameters are exhibited in Table 2. Mechanical behaviours (e.g. friction and dilatation) on coal cleats produce a significant impact on stress distribution, crack behaviour and strength anisotropy of the coal model.

Based on the micro-structural parameters in Table 1 and the parameters for the coal cleat system in Table 2, macro mechanical properties of the coal model are approaching real coal samples (referring to a real coal sample with uniaxial compressive strength $\sigma_{uc} = 7.5$ MPa, Young’s modulus $E = 3.5$ GPa, and Poisson’s ratio $\nu = 0.48$). The mechanical properties of the coal model can be described as follows. Uniaxial loading along the “x” axis gives the properties: $\sigma_{uc} = 18.3$ MPa, $E = 3.8$ GPa and $\nu = 0.41$; along the “y” axis, $\sigma_{uc} = 17.1$ MPa, $E = 3.3$ GPa and $\nu = 0.43$; along the “z” axis, $\sigma_{uc} = 18.8$ MPa, $E = 3.3$ GPa and $\nu = 0.41$. The anisotropy of macro mechanical properties are formed because of the existence of the coal cleat system.

3.2.2. (2) Pressure schemes

PHF simulation is carried out based on the calibrated coal model. The basic ideas and procedures of numerical simulation are partially borrowed from Li’s PHF experimental system (Li et al., 2014). Fig. 4b shows the sketch of PHF numerical simulation. Six walls are set up to confine the coal model as the tri-axial preloading system and energy absorbing boundaries. 10-MPa preloading is applied to simulate the original crustal stresses in the coal reservoir, which enhances the original mechanical properties of the coal model. The hydraulic pressure stimulated by PHF is loaded in a 20.0-mm cubic space (fracturing space) at the centre of the coal model. A fracturing pipe with the 500-mm length is supposed to extend to the fracturing space and aids to provide the analytical solution of the input pressure.

To stimulate pressure ripples at the centre of the model, the hydraulic pressure boundary (not the injection rate boundary) is applied within the fracturing space’s domains. Three input schemes of hydraulic pressure are proposed in comparison to seeking the mechanism of PHF stimulation (one cycle is illustrated in Fig. 5):

- Scheme I is proposed to simulate the traditional HF, with a constant hydrostatic pressure of 22 MPa.
- Scheme II is a sinusoidal pressure ripple (20-Hz frequency, 20-MPa average pressure, and 2-MPa amplitude). It is proposed to study the fracturing effect without high-frequency components.
- Scheme III is obtained from the analytical solution, by using scheme II as input function $f(t)$, with ten orders of high-frequency components. It is aiming to study the actual fracturing effect after propagation of the sinusoidal pressure ripples along the fracturing pipe. Correlative parameters used to generate scheme III are detailed in Table 3.

When scheme II or scheme III is applied in the model, a dynamic solution (which is the default behaviour of PFC3D) is fully activated. The full dynamic equations of motion including inertial terms are solved. The generation and dissipation (by the default damping and frictional behaviours) of kinetic energy are considered in the dynamic solution. The pressure ripples are implemented at each time step after the motion calculation via a “Fishcall” function.

To clearly discuss the fracturing effect by pressure ripples, a limited process of seepage (by setting a small flowing step of $10^{-4}$ s) is designed to strengthen the mechanical effect of pressure ripples (i.e. dynamic action on the model) and weaken the effect of fluid flows (flow range is limited around the fracturing space). The time duration for all schemes is 2.8 s (including 56 cycles of pressure ripples).

3.3. PHF simulation and validation

The next is to simulate and clarify the fracturing mechanism of PHF (scheme III) by comparing with the other two schemes. Various advanced indicators are introduced to assess the fracturing effects and validation of PHF simulation is discussed later.

3.3.1. (1) Crack propagation

The fracturing process is monitored and discussed by various indicators. The particle velocity stimulated by PHF is fundamental for crack propagation. Velocities (located in top, bottom, left and right) around the fracturing space are measured in scheme II and III (velocity fluctuation is negligible in scheme I). Constant vibrations

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![Fig. 4. Sketch of PHF simulation in coal model.](image-url)
of particles inside the coal model can be recorded due to the stimulation of PHF pressure ripples (Fig. 6). These movements can result in cracking developments. Particle velocities in scheme II have a more distinct sinusoidal tendency due to the relatively simple injecting function, while the counterparts in scheme III are changing more drastic and irregular because the high-frequency components from the analytical solution are introduced.

Distributions of cracks and contact forces inside the coal model, stimulated by the three input schemes, are exhibited in Fig. 7 (red dots denote tensile cracks, blue dots denote shear cracks and black lines denote contact forces between particles). It can be seen that the contact forces are more concentrated (denoted by the thicker lines) around the fracturing space under stimulation of the pressure ripples. The coal model stimulated by a constant water pressure (scheme I) only developed a single dominating fracture, with branch fractures being nearly negligible (Fig. 7a). However, simulation results in scheme II and III indicate that both main fractures and branch fractures are fully developed (Fig. 7b and c). The sectional views of particle clusters (identified by diverse colours) are exhibited at the lower-right corner, indicating similar phenomena observed from crack propagations. From scheme I to III, discrete particles and clusters are increased. The original cleats intersect with each other and more clusters are produced under pressure ripples.

Significantly, a new type of fracture i.e., ‘spacing fracture’ is generated by the pulse loading. Fig. 7d shows the plan view of fractures stimulated by scheme III. The spacing fractures are produced around the main fracture (resemble the branch fractures) but are not attached to it. The formation of spacing fractures can be

### Table 1
Calibrated parameters of bond and particle in coal model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( R_{\text{max}} ) (mm)</th>
<th>( R_{\text{min}} ) (mm)</th>
<th>( \psi )</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( E_p ) (GPa)</th>
<th>( E_c ) (GPa)</th>
<th>( k_n/l_s )</th>
<th>( k_l/l_s )</th>
<th>( \sigma_c ) (MPa)</th>
<th>( \Delta \sigma_c ) (MPa)</th>
<th>( \tau_c ) (MPa)</th>
<th>( \Delta \tau_c ) (MPa)</th>
<th>( \mu_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>6.5</td>
<td>4.6</td>
<td>12%</td>
<td>20.5</td>
<td>5.5</td>
<td>6.2</td>
<td>3.5</td>
<td>3.1</td>
<td>20</td>
<td>4</td>
<td>25</td>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: \( R_{\text{max}} \)-Maximum particle radius; \( R_{\text{min}} \)-Minimum particle radius; \( \psi \)-Porosity; \( \rho \)-Particle density; \( E_p \)-Young’s modulus of particle; \( E_c \)-Young’s modulus of parallel bond; \( k_n/l_s \)-Normal/Shear stiffness ratio of particle contact; \( k_l/l_s \)-Normal/Shear stiffness ratio of parallel bond; \( \sigma_c \)-Tensile strength of bond; \( \tau_c \)-Shear strength of bond; \( \Delta \sigma_c \), \( \Delta \tau_c \)-Standard deviation of tensile strength and shear strength; \( \mu_p \)-Frictional coefficient of particle contact.

### Table 2
Parameters for the three sets of coal cleat.

<table>
<thead>
<tr>
<th></th>
<th>( k_n ) (GPa/m)</th>
<th>( k_l ) (GPa/m)</th>
<th>( \psi ) (%)</th>
<th>( \mu_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bedding Plane</td>
<td>40</td>
<td>40</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>Face Cleat</td>
<td>30</td>
<td>30</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>Butt Cleat</td>
<td>30</td>
<td>30</td>
<td>6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: \( k_n \)-Normal stiffness of smooth joint; \( k_l \)-Shear stiffness of smooth joint; \( \psi \)-Dilatational angle; \( \mu_c \)-Frictional coefficient of smooth joint contact.

### Table 3
Calculation parameters for scheme III.

<table>
<thead>
<tr>
<th>( L ) (m)</th>
<th>( \rho_w ) (kg/m(^3))</th>
<th>( \lambda ) (GPa)</th>
<th>( \nu ) (rad/s)</th>
<th>( f ) (Hz)</th>
<th>( T ) (s)</th>
<th>( \Delta P_a ) (MPa)</th>
<th>( P_{\text{average}} ) (MPa)</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1000.0</td>
<td>0.5</td>
<td>2.2</td>
<td>1.5</td>
<td>40( \pi )</td>
<td>20</td>
<td>2.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Fig. 5. Three input schemes of applied hydraulic pressure.
explained in two aspects: bond breaking between particles and the intersection of smooth joints (e.g., the No. 1 and No. 2 intersections in Fig. 7d), which indicates that the generation of this type of fracture is highly affected by the original coal cleat system. As the spacing fractures and branch fractures may gradually merge together with the accumulation of cracks, spacing fractures can also be viewed as a developmental stage for branch fractures. Obviously, this type of fracture improves the extension of branch fractures so that a better fracture network can be formed from the PHF effect. Field PHF test exhibits similar fracture networks with the simulation results (Fig. 8). The fully developed spacing fractures and branch fractures extend the fracture network, which greatly increases the permeability of coal reservoirs.

The spacing fracture is a unique fracture type from the PHF stimulation. It is not directly resulted from the increase of water pressure but rather generated by the energy of pressure ripples that are conducted from the fracturing fluid into the surrounding coal mass. Pressure ripples continuously pound the particles and form serials of excitation so that the wave energy can be transported into the coal mass. During the constant stimulation, cracks are gradually
3.3.2. (2) Development of hydraulic pressure

Development of hydraulic pressure (in scheme III) approaching the fracturing space is recorded in Fig. 9 and two unbroken hosting domains for hydraulic pressures (H1 & H2 in Fig. 4b) are selected to illustrate. Firstly, hydraulic pressures are increasing in vibration and the vibration becomes more intensive when hydraulic pressure grows larger. This is caused by a hydraulic-mechanical coupling effect of the second term in equation (12), which represents that the mechanical change in domain volume causes a change in domain pressure, stimulated by the periodic loading of pressure ripples. The peak pressure in vibration necessarily has a major impact on crack behaviours.

Considering the PHF effects and the overall dynamic stability of the coal model, the seepage range is set to be confined to the fracturing space (refer to the last paragraph in 3.2). The strengthened crack behaviours caused by pressure vibration in domains are believed to exist close to the fracturing space. Moreover, the combined dynamic actions of the pressure ripples (from the fracturing space) and the pressure vibrations (from the flow process) are much complicated and need more consideration in future works.

3.3.3. (3) Fracturing process and energy evolution

Strain energy stored between particles will be released when cracking occurs. The indicator of “crack intensity” is proposed to measure the energy release so that fracturing effects can be properly discussed. In laboratory experiments, this type of energy is detected by using an acoustic energy (AE) monitor that is attached to the surface of the rock sample. In this paper, crack intensities (assumed to be the complete release of the peak strain energy) are directly measured and located inside the coal model, which can be calculated based on:

\[
E_s = \frac{f_2^2}{2k_n} + \frac{f_2^2}{2k_s},
\]

where \(E_s\) denotes the strain energy; \(f_n\) and \(f_s\) denote the normal and shear contact forces, respectively; \(k_n\) and \(k_s\) denote the normal and shear stiffness, respectively.

Besides, advanced indicators are proposed to quantify the fracturing effects in simulation. Crack events density (CED) denotes the event counts at each calculation step and has a proportional relationship with crack intensity rate (CIR). CIR denotes the result of average crack intensity plus event counts at each calculation step. Kinetic energy (KE) is measured as the summation of all particles’ kinetic energy in the coal model:

\[
E_k = \frac{1}{2} \sum_{i \in N_0} m_i \mathbf{V}_i^2 + I_i (\omega_i^2),
\]

where \(N_0\) denotes a number of particles, \(m_i\) denotes the inertial mass, \(I_i\) denotes the inertial vector, \(\mathbf{V}_i\) denotes the average velocity and \(\omega_i\) denotes the rotational velocity. Because of the inclusion of all particles, KE can accurately detect the fluctuation and accumulation of energy for the whole model. These three indicators CED, CIR and KE can effectively recognize the characteristics of crack events over the time history (or the progress of calculation steps).

Figs. 10a, 11a and 12a exhibit the evolutions of indicators by the three pressure schemes. In scheme I, the crack events and KE only appear at the early stage, which indicates that the coal model enters a stable state after a short fracturing duration. However, in PHF schemes II and III, CED and CIR constantly develop and present relatively larger values throughout the whole fracturing process. KE evolutions have a continuous fluctuating pattern due to pressure ripples and can be clearly distinguished and divided into four stages: ① initial disturbing stage ② continuous developing stage ③ unstable stage ④ post-unstable stage. The initial disturbing stage represents the early fracturing, during which kinetic energy accumulates and shows a drastically changing pattern. In the continuous developing stage, cracks are constantly generated inside the coal model and KE presents a stably changing pattern. When the whole coal model ruptures after the accumulation of cracks, this is going through the unstable stage; hence, the changing patterns of CED, CIR, and KE become unstable. The post-unstable stage characterizes the behaviour of the coal model after the rupture with distinctly lower KE, indicating that the coal model loses the capability to receive energy from pulse loading. Previous laboratory experiments present similar energy evolution with the simulation results (Li et al., 2013; Zhai et al., 2015). According to the results from Li et al. (2013), the AE energy rate monitored in the sample exhibits as the same developmental stages as the KE in the simulation.
Figs. 10b, 11b and 12b exhibit the spatial distributions of crack intensity by the three schemes. The maximum energy intensity of the three schemes is quite close. Compared with the other two, scheme III developed a higher scattered distribution and the energy can be detected relatively far away from the center of the coal model. Although the distributions of crack events in scheme II and...
scheme III are similar over the time history, the fracturing effects inside the coal models are quite different. CED and CIR in scheme III are higher than in scheme II, which indicates the stronger fracturing effects of scheme III. The higher KE can be observed in scheme III than in scheme II for all stages. The unstable stage in scheme III occurs a bit earlier than in scheme II. All of these phenomena are resulting from the high frequency components of pressure ripples and stronger cracking is produced during simulation.

4. Parametric effects of PHF and discussion

According to the above sections, the analytical solution for pressure ripples is obtained and the fracturing mechanism of PHF is discussed for different pulsating schemes. The simulation results indicate that pressure ripples stimulated by PHF have a unique fracturing effect on the coal mass. Particularly, the high frequency components, which are crucial but ruled out from the common understanding of pressure propagation, have a strong impact on the fracturing process and mass particles’ behaviours. In this section, fracturing effects affected by different PHF parameters are studied based on the scheme III. Two important parameters i.e., frequency and amplitude, which strongly affect the fracturing process, are tested and discussed. These parameters are also relatively convenient to be controlled in actual engineering applications.

4.1. Fracturing effects by different frequencies

This study considers three different PHF frequencies (20 Hz, 25 Hz, and 30 Hz, which are equivalent to 56, 70 and 84 cycles of pressure ripples, respectively) with the same ripple amplitude. Fig. 13 indicates that under the same average pressure (20 MPa) and ripple amplitude (2 MPa) (based on the input function $f(t)$), higher frequency of PHF can result in a larger expanded fracture network (fracturing result for the case of 20 Hz can be found in Fig. 7c);
particularly, more spacing fractures can be developed. It can also be observed from Fig. 12a and Fig. 14 that crack events appear more frequently over time history with the increase of pulse frequency; however, the maximum values of CED and CRI are very close in each case. The values of KE indicate that the increase of pulse frequency mainly affect the fracturing progress in the continuous developing stage, during which a higher frequency results in a higher vibrating pattern of kinetic energy.

Although more cracks are developed under higher pulse frequencies, the whole progress is not accelerated and the unstable and post-unstable stages accrue approximately at the same time. The simulations conclude that relatively higher frequency can produce a better fracturing effect in actual applications but may not shorten the fracturing duration. The results do not necessarily indicate that very high frequency should be used because the engineering efficiency needs to be taken into consideration.

4.2. Fracturing effects by different amplitudes

The amplitude of pressure ripples, which can be directly controlled by the piston pump of the PHF device, is also a crucial factor for fracturing effects. Similarly, the study (based on scheme III) numerically simulates fracturing effects under three different amplitudes ($\Delta P_a = 2$ MPa, 3 MPa and 4 MPa) with the same frequency. Fig. 15 indicates the fracturing results with the increase of amplitude: the fractured areas become more concentrated, the number of spacing fractures are largely decreased and the distribution of cracks are not obviously expanded (fracturing results for the case of $\Delta P_a = 2$ MPa can be found in Fig. 7c).

Moreover, Figs. 12a and 16 indicate that with the increase of ripple amplitudes, the magnitudes of crack events (CED and CIR) are obviously increasing; however, crack events appear less frequently compared with the records in Fig. 14 throughout the time history. The increase of amplitude also affects the continuous developing stage, during which KE is increasing with higher amplitude. More importantly, KE evolutions show that the unstable

Fig. 15. Distributions of cracks and contact force under different amplitudes.

Fig. 16. Fracturing process and developments of CED, CIR, and KE.
and post-unstable stages appear earlier in time history, which indicates that relatively higher amplitude may accelerate the rupturing progress of the coal model and make the fracturing duration shorter. Note that higher ripple amplitudes can effectively accelerate the rupturing progress and shorten the fracturing time. However, a better fracture network does not necessarily to be produced and spacing/branch fractures are not necessarily developed.

5. Conclusions

In this study, an advanced mathematical model for pressure ripple propagation is proposed and an analytical solution is obtained. Unlike traditional understanding, which believes that pulsating hydraulic fracturing (PHF) creates simple sinusoidal pressure ripples inside a fracturing pipe, an actual pressure changing pattern is more drastic with high-frequency components. In laboratory experiments, the high-frequency components are naturally produced because of the strong wave impedance materials; this wave form can be generated by using ‘seal-release’ pulsating scheme. Eventually, this solution is applied for the pulsating loadings in numerical simulations.

Based on the discrete element method, hydraulic fracturing (HF) and PHF fracturing effects are numerically simulated in comparative situations. The simulation results indicate that compared with the traditional HF loading scheme, the schemes with pressure ripples can create a better distribution of cracks and fractures. These unique effects can be amplified by introducing the high-frequency components of ripples. The pulsating energy propagates through a fracturing pipe and enters into coal mass to produce a new type of fracture (i.e., spacing fracture). This type of fracture can intersect with branch fractures and original cleats of the coal mass and result in a well-distributed fracture network. Advanced indicators that include crack event density (CED), crack intensity rate (CIR) and kinetic energy (KE) are proposed and measured to clarify the conclusions of PHF effects. Particularly, the KE evolution helps to divide the development of fractures under PHF into four stages (I: initial disturbing stage, 2: continuous developing stage, 3: unstable stage, 4: post-unstable stage).

Furthermore, fracturing effects with different parameters (i.e., pulsating frequency and ripple amplitude) are simulated and discussed for engineering optimization. Simulation results suggest that relatively higher pulsating frequency can stimulate a better-distributed fracture network; however, the fracturing duration may not be significantly shortened by this strategy. Higher ripple amplitude can accelerate the fracturing progress and rupture of the coal model (unstable and post-unstable stages) appears earlier with higher amplitudes. However, this phenomenon may have negative effects on the distribution and extension of a fracture network.

Funding

This research was supported by National Natural Science Foundation of China (Grant No. 41230635) and the open fund (Grant No. SKLG20152004) from State Key Laboratory of Geo-hazard Prevention and Geo-environment Protection, China.

Acknowledgements

The authors would like to thank all of the members in ‘Multi-scale, Multiphysics Modelling’ (M3) group of the Centre for Geoscience Computing, University of Queensland; this research could not be completed without their valuable assistance and encouragement.

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