Reliability Analysis of Circular Tunnel Face Stability Obeying Hoek–Brown Failure Criterion Considering Different Distribution Types and Correlation Structures

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Abstract: A reliability analysis is presented of a circular tunnel face driven by a pressurized shield in a highly fractured Hoek–Brown (HB) rock mass. A limit analysis approach is employed to define a limit state function (LSF) based on an advanced rotational mechanism. The objective is to analyze how different reliability methods, different assumptions about distribution types and correlation structures of the random variables, and different tunnel sizes and support pressures influence the computed reliability results. Results indicate that the reliability method has a limited influence on reliability results, hence suggesting that the LSF is not highly nonlinear and that a computationally efficient method can be employed; and that all random variables under consideration are resistance variables, with their assumed distribution types and correlation structures having a significant effect on the reliability results. This emphasizes the importance of an adequate characterization of geotechnical uncertainties for practical applications. This also confirms that the reliability of tunnel face stability increases significantly as the face support increases or the tunnel diameter decreases. DOI: 10.1061/(ASCE)CP.1943-5487.0000464. © 2014 American Society of Civil Engineers.

Author keywords: Tunnel face stability; Hoek–Brown; Reliability analysis; Response surface method; First-order reliability method; Sensitivity.

Introduction

Geological and geotechnical uncertainties always exist in real projects, because geological data are obtained from imperfect site investigations, and geotechnical models and parameters are uncertain. Deterministic models cannot adequately address this uncertainty (Ching et al. 2009), so reliability methods have become a rational alternative to consider the effects of uncertainties on engineering designs.

Circular tunnels excavated with a pressurized shield are common in projects related to, for instance, metro, water supply, and sewage facilities. The face pressure is an important parameter to avoid face collapses in these tunnels, and several methods for analysis have been proposed (Leca and Dormieux 1990; Mollon et al. 2010; Senent et al. 2013). In recent years, there has been increasing interest in the reliability aspects of tunnel face stability; for instance, Mollon et al. (2009a, b, 2013) studied the failure probability of tunnel faces in soils with the Mohr-Coulomb failure criterion by using the first-order reliability method (FORM) and the response surface method (RSM).

This paper presents a reliability analysis—using FORM, RSM, and importance sampling (IS)—of a circular tunnel face driven by a pressurized shield in a highly fractured Hoek–Brown (HB) rock mass. The objective is to analyze how different assumptions about distribution types and correlation structures affect the reliability results. In addition, with the purpose of identifying the most relevant variables for engineering design, this study investigates the sensitivity of the reliability index to changes in the random variables involved for different support pressures. The influence of tunnel size on the reliability index is also studied.

Deterministic Model

This study employs the tunnel face failure mechanism recently proposed by Senent et al. (2013) in reliability computations. The model is an upper-bound solution in the context of limit analysis, which extends the mechanism presented by Mollon et al. (2011b) to account for nonlinear failure criteria such as the HB criterion. The mechanism comprises a unique block that rotates around an axis perpendicular to the vertical plane of symmetry of the tunnel cross section. The contours of the mechanism in this vertical plane of symmetry are approximately logarithmic spirals, and the slip surface affects the whole excavation front (Fig. 1).

To construct the mechanism, the perimeter of the tunnel face is discretized first, producing a set of \( n_i \) points \( A_i \) and \( A_i' \), for \( j = 1 \) to \( n_i/2 \), as shown in Fig. 1(a). Then, a set of planes is defined that contain the mechanism’s rotation axis: the first part [Section 1 in Fig. 1(b)] corresponds to planes that intersect the tunnel face, with each plane containing two of these \( A_i \) and \( A_i' \) points; in the second part of the mechanism (Section 2), each plane rotates an angle \( \delta \) with the previous plane. That is, the mechanism is defined by two unique parameters, \( n_i \) and \( \delta \), so that the precision of the mechanism increases as \( n_i \) increases and \( \delta \) decreases.

Once the discretization is completed, a point-to-point slip surface of the mechanism can be constructed. To that end, triangular facets, \( F_{i,j} \), are constructed as follows (Fig. 2): two points that belong to plane \( \Pi_j \) (\( P_{i,j} \) and \( P_{i+1,j} \)) are used, from which a new
point \((P_{j+1})\) is computed in plane \(\Pi_{j+1}\), so that the slip surface defined by such three points follows the associated flow rule required by limit analysis. Fig. 3 shows an example of slip surface obtained with this procedure. Further details are available in Mollon et al. (2011b) and Senent et al. (2013).

Once the failure mechanism has been defined with \(F_{i,j}\) facets, the collapse pressure can be computed by using the upper bound theorem of limit analysis—i.e., equating the energy applied into the system with that dissipated by the system. However, such collapse pressure corresponds to a specific position of the mechanism’s axis of rotation (defined by \(\omega_E\) and \(r_E\), as shown in Fig. 1), so it is necessary to maximize it in relation to \(\omega_E\) and \(r_E\) to obtain the "optimum" upper-bound solutions. In addition, to compute dissipated energies, the (equivalent) friction angle and cohesion are needed for all points (or facets) along the slip surface. With a traditional linear Mohr-Coulomb criterion, they are constant; for nonlinear criteria, equivalent cohesion and friction angle values can be computed by using local approximations (tangents) to the real failure criterion at adequate stress levels, hence introducing the need to compute the stresses acting along the slip surface. As described by Senent et al. (2013), results from three-dimensional finite-difference simulations with 

\[ \text{FLAC3D} \]

suggest that considering a linear stress distribution is adequate in most practical applications. As a result, the best upper-bound solution is obtained through maximizing with respect to two geometrical variables (that define the axis of rotation) and two stress variables (that define the stress distribution along the slip surface).

### Performance Function and Uncertainty Characteristics

#### Performance Function

Following the standard convention from the reliability literature, the performance function for tunnel face collapse can be defined as

\[
G(X) = \frac{\sigma_t}{\sigma_c(X)} - 1
\]

where vector \(X\) = random variables considered in the model; \(\sigma_t\) = support pressure applied at the tunnel face; and \(\sigma_c(X)\) = collapse pressure provided by the advanced rotational mechanism \(\sigma_c(X)\) values computed with the proposed mechanism are always nonnegative, because the limit-analysis code has been set to report \(\sigma_c(X) = 0\) for stable tunnel faces. This avoids the numerical problem that may arise when negative values are generated by using Monte Carlo simulations (MCS) (Low 2010).

#### Characterization of Uncertainty: Deterministic and Random Variables

For simplicity, the unit weight of the rock mass, the disturbance value employed in the HB criterion, and the circular tunnel diameter are considered deterministic, with values of 24 kN/m³, 0, and 10 m, respectively. These values define the base case in this study. Because the applied support pressure is normally well controlled in modern tunnel-boring machines (TBMs), it is also considered as a deterministic variable, although given its importance, a sensitivity analysis is conducted by using different values of \(\sigma_t\).

Therefore, only the geological parameters that affect the rock strength provided by the HB criterion—the uniaxial compressive strength of intact rock, \(\sigma_{ci}\); the rock mass quality given by GSI; and
the $m_i$ parameter of the HB failure criterion—are considered as random variables. To use a realistic set of parameters as an example, this study uses information published in the literature about weak rock masses, in which face instability problems can be expected. For instance, Hoek et al. (1998) and Marinos and Hoek (2000) provided the parameters of a black shale, with estimated interval values of $\sigma_{ci} = 1$–5 MPa, $\text{GSI} = 15 \pm 8$, and $m_i = 6 \pm 2$. Assuming that the three variables obey the “three sigma rule” (Duncan 2000) — which means that nearly all values lie within three SDs of the mean in a normal distribution — the mean values ($\mu$) and coefficients of variation (COVs) listed in Table 1 can be obtained. [However, although the use of the three sigma rule is appropriate given the small amount of available data, it does not guarantee the adequacy of distribution tails, which may affect the probability of failure in some cases (Jimenez and Sitar 2009). Therefore, this may become a topic of research in future extensions of this work.]

In addition, to illustrate the influence of the types of statistical distributions and of the correlation structures on the reliability results, four cases are considered:

1. Normal distributions and uncorrelated random variables;
2. Nonnormal distributions and uncorrelated random variables;
3. Normal distributions and correlated random variables; and
4. Nonnormal distributions and correlated random variables.

In Cases 1 and 3, all random variables ($\sigma_{ci}$, GSI, and $m_i$) are assumed to follow normal distributions with the mean values and coefficients of variation listed in Table 1. For nonnormal distributions (i.e., Cases 2 and 4), because the values of $\sigma_{ci}$ and $m_i$ should be nonnegative, $\sigma_{ci}$ and $m_i$ are both assumed to be lognormal; similarly, because the rock mass is highly fractured, GSI is assumed to follow a (bounded) beta distribution within the [0, 25] interval, as listed in Table 1.

Similarly, two cases are considered for the correlation structure: Cases 1 and 2 consider uncorrelated variables, so that all correlation coefficients among $\sigma_{ci}$, $m_i$, and GSI are zero, whereas Cases 3 and 4 consider correlated variables. In this case, it is difficult to define correlation coefficients, such as $\rho_{\sigma_{ci},\text{GSI}}$, based on published data only. However, approximate relationships can be applied, together with MCS, as a guidance to evaluate $\rho_{\sigma_{ci},\text{GSI}}$. Hoek and Brown (1997) suggested that $\text{GSI} = \text{RMR} - 5$ for the 1989 version of Bieniawski’s rock mass rating (RMR) classification (Bieniawski 1989), where RMR is computed by summing the evaluations of five parameters (strength of the rock, drill core quality, groundwater conditions, joint and fracture spacing, and joint characteristics). Because rock strength can be expressed by $\sigma_{ci}$, an approximated relationship between GSI and $\sigma_{ci}$ can be built. To conduct MCS, all parameters (other than joint characteristics) are assumed to follow uniform distributions; for the joint characteristics, its five conditions are sampled with equal probability. After conducting 10,000 simulations, $\rho_{\sigma_{ci},\text{GSI}}$ is found to be approximately 0.3, which will be used in the present study.

The correlation coefficient between $\sigma_{ci}$ and $m_i$ is also difficult to estimate, although more data are available in this case. For instance, the RocData 4.0 database provides several data groups of $\sigma_{ci}$ and $m_i$ for different types of rock (such as coal, limestone, and sandstone). These data groups are employed to estimate a value of $\rho_{\sigma_{ci},m_i}$ in the current analysis. In particular, this study considers six rock

## Table 1. Statistical Properties of Random Variables Characterizing Rock Mass Strength

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution type</th>
<th>Parameter</th>
<th>Distribution type</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{ci}$</td>
<td>Normal</td>
<td>3</td>
<td>0.22</td>
<td>—</td>
</tr>
<tr>
<td>(MPa)</td>
<td>Lognormal</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>GSI</td>
<td>Normal</td>
<td>15</td>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Normal</td>
<td>6</td>
<td>0.11</td>
<td>25</td>
</tr>
<tr>
<td>Lognormal</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The lower and upper bounds are defined for beta distribution only.*
In engineering practice, the joint probability density function of all random variables. The transformation can be expressed as

\[ \sigma \text{relation between } \] with Shen and Karakus (2014), this suggests that a negative correlation between variables under consideration is not easy to acquire. However, because the marginal distribution and the correlation relationship are known, the Nataf transformation (Liu and Der Kiureghian 1986) is used to transform the variables into uncorrelated, N(0,1) random variables.

The components of vector \( Z \) have zero mean and unit SD, with a correlation matrix between the \( Z_i \) variables given by

\[ R' = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} \] (3)

where elements \( \rho_{ij}' \) are solution of the integral equation

\[ \rho_{ij} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x_i - \mu_i)(x_j - \mu_j)}{\sigma_i \sigma_j} \varphi_2(z_i, z_j, \rho_{ij}')dz_i dz_j \] (4)

where \( \rho_{ij} \) is correlation coefficient of the original variables in Table 2, and \( \varphi_2 \) is the joint probability density function (PDF) of two N(0,1) random variables with correlation coefficient given by \( \rho_{ij}' \). Liu and Der Kiureghian (1986) gave an approximation solution for \( \rho_{ij}' \), but also indicated that the difference between \( \rho_{ij} \) and \( \rho_{ij}' \) is negligible. Therefore, in this study, it is assumed that \( \rho_{ij}' = \rho_{ij} \) (equivalent to \( R' = R \)) for the cases with nonnormal variables. Further information on how to compute the true value of \( \rho_{ij}' \) is discussed by Liu and Der Kiureghian (1986).

Finally, the correlated variables, \( Z \), can be transformed into uncorrelated variables, \( U \), using the Cholesky factorization as

\[ U = L^{-1}(Z)^T \] (5)

where the lower triangular matrix, \( L \) = Cholesky factor of \( R' \), and \( R' = LL^T \). \((T\) stands for the transpose matrix operator.\)

### Reliability Analysis

**General Formulation**

Reliability methods aim to compute the probability of failure by the following integral:

\[ P_f = P(G(X) \leq 0) = \int_{G(X) \leq 0} f_X(X) dX \] (6)

where \( G(X) = \text{limit state function in physical space } \) \( |G(X)| \leq 0 \) indicates failure, and \( f_X(X) = \text{joint probability density function (PDF) of input random variables, } X \). Although direct resolution of Eq. (6) in the physical space is possible (Low and Tang 1997), to ease the solution process and the interpretation of results, it is interesting to transform the original vector, \( X \), into the standard normal space of uncorrelated, \( N(0,1) \), random variables. As illustrated by Der Kiureghian (2005), Eq. (6) can then be rewritten as

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Table 2. Matrix of Correlation Structure, \( R \), for Cases 3 and 4 with Correlated Variables

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \sigma_{ci} )</th>
<th>GSI</th>
<th>( m_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{ci} )</td>
<td>1</td>
<td>0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>GSI</td>
<td>0.3</td>
<td>1</td>
<td>-0.25</td>
</tr>
<tr>
<td>( m_i )</td>
<td>-0.4</td>
<td>-0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ Z_i = \Phi^{-1}[F_{X_i}(X_i)], \quad i = 1, 2, \ldots, n \] (2)

where \( F_{X_i} = \text{cumulative distribution function (CDF) of the original random variable, } X_i, \) and \( \Phi(\cdot) = \text{standard normal CDF} \). The elements of vector \( Z \) have zero mean and unit SD, with a correlation matrix between the \( Z_i \) variables given by

\[ R' = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} \] (3)

where elements \( \rho_{ij}' \) are solution of the integral equation

\[ \rho_{ij} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x_i - \mu_i)(x_j - \mu_j)}{\sigma_i \sigma_j} \varphi_2(z_i, z_j, \rho_{ij}')dz_i dz_j \] (4)

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Finally, the correlated variables, \( Z \), can be transformed into uncorrelated variables, \( U \), using the Cholesky factorization as

\[ U = L^{-1}(Z)^T \] (5)

where the lower triangular matrix, \( L \) = Cholesky factor of \( R' \), and \( R' = LL^T \). \((T\) stands for the transpose matrix operator.\)
\[
P_f = P[g(U) \leq 0] = \int_{g(U) \leq 0} f_U(U) dU
\]  

where \(U = \) vector of standard normal and uncorrelated random variables, \(u_i (i = 1, 2, \ldots, n); \) \(f_U(U) = \) joint PDF of \(U; \) and \(g(U) = \) (transformed) limit state function (also known as performance function).

However, because computing direct estimates of the integral given by Eq. (7) is challenging, and because the employed reliability method may affect the reliability results, alternative procedures such as FORM, RSM, and IS are employed herein to compute the reliability and probability of failure. Also, sensitivities are computed to illustrate which input random variables have stronger effects on the computed reliability results.

**Reliability Methods**

**First-Order Reliability Method**

The FORM was initially proposed by Cornell (1969), and later extended by Hasofer and Lind (1974), Rackwitz and Flessler (1978), and Zhang and Der Kiureghian (1995). FORM uses a linear approximation as

\[
g(U) \approx L(U) = g(u) + \nabla g(u)(U - u)^T
\]

where \(L(U) = \) linearized performance function; \(u = \) expansion point; and \(\nabla g(u) = \) gradient of \(g(U) \) at \(u\).

To minimize the error, expanding \(g(U) \) at the point with the highest probability density—called the design point, \(u^*\)—is the best choice. This is equivalent to finding a point on the limit state surface with the shortest distance to the origin in \(U\)-space. Therefore, the design point can be obtained by solving the following constrained optimization problem:

\[
u^* = \min_u \|u\| \quad \text{subject to} \quad g(u) = 0
\]

where \(\| \cdot \| \) = norm of a vector. Then, the reliability index can be computed as

\[
\beta = \|u^*\|
\]

and the probability of failure can be approximated as

\[
P_f \approx \Phi(-\beta)
\]

**Response Surface Method**

By selecting deterministic experimental points, the RSM aims to construct a closed-form polynomial function, called the response surface (RS), which is approximately equal to the actual limit state function (LSF) in the region with maximum contribution to the probability of failure. Reliability analysis can then be easily performed based on this RS.

An iterative RSM proposed by Rajashekkar and Ellingwood (1993)—which is widely employed in tunnel reliability analysis (Lü and Low 2011; Lü et al. 2011; Mollon et al. 2009b)—is employed in this study to obtain the RS and the corresponding design point. The quadratic polynomial function without cross terms in \(U\)-space is defined as

\[
g(u) = a_0 + \sum_{i=1}^n b_i u_i + \sum_{i=1}^n c_i u_i^2
\]

where \(a_0, b_i, \) and \(c_i = \) coefficients to be determined using the equation provided by performances computed at the sampling points; and \(n = \) number of random variables under consideration.

**Importance Sampling**

Given the specific nature of simulation methods, the RS \(\tilde{g}(u)\) does not need to reproduce the exact LSF \(g(u)\) in the entire space, but only in the region near the actual design point, \(u^*\), which contributes most to the total failure probability. Thus, the IS (Schuëller and Stix 1987) is particularly suited to efficiently compute—primarily sampling in the area near the design point—the failure probability, as follows:

\[
P_f = \frac{1}{m} \sum_{i=1}^m \frac{I[\tilde{g}(u_i)]}{h(u_i)}
\]

where \(m = \) number of samples, and \(h(u) = \) importance sampling joint PDF.

**Reliability Approaches Employed in This Work**

Three different approaches will be used in this work to analyze the reliability of tunnel face stability. They are

1. Response surface method with first-order reliability method (RSM-FORM), in which FORM is applied to the RS that approximates the LSF;
2. Response surface method with importance sampling (RSM-IS), in which IS is applied to the response surface in the vicinity of the design point computed with FORM for such RS; and
3. First-order reliability method with the real LSF (LSF-FORM).

In Fig. 5, a cubic polynomial is used to illustrate the differences between these three approaches by using an example in two-dimensional \(U\)-space. Design Point 1 is found by using Approach 3; i.e., using FORM directly on the LSF. The RS employed in Approaches 1 and 2 is an approximate surface, obtained with the previously mentioned iterative RSM, which is very close to the LSF in the region that contributes primarily to the probability of failure. The difference between them is associated with how the reliability problem is solved in both cases: in Approach 1, a FORM approximation is employed, using such approximated RS, to find Design Point 2 and its associated probability of failure and reliability index; whereas, in Approach 2, IS is applied to the RS near Design Point 2. It is obvious that the RSM-IS approach is more accurate than the RSM-FORM approach, because the latter is based on a linear approximation of the RS at Design Point 2. It is also reasonable to expect that Design Point 1 is more accurate than Design Point 2, because Design Point 1 is calculated based on the true LSF, whereas Design Point 2 is based on an approximated surface.

**Sensitivity Analysis**

A sensitivity analysis indicates the sensitivity of the reliability index to changes in the input random variables. It also provides the order of importance of the random variables, playing an important role in many applications of reliability-based design. Der Kiureghian (2005) described an approach to compute sensitivity. More recently, Chan and Low (2012) suggested a simpler algorithm, given by

\[
\gamma = \frac{\partial \beta}{\partial X} \approx \frac{\partial \beta}{\partial \mathcal{Z}} = (L^{-1})^T \alpha
\]

where \(\alpha = u^*/\|u^*\| = \) unit design point vector, provided by FORM.
Comparison of Different Computational Approaches

The three previously discussed approaches (RSM-IS, RSM-FORM, and LSF-FORM) are used to compute the reliability indexes, \( P_f \), of tunnel faces designed with the deterministic parameters and the previously indicated uncorrelated normal variables. [When the shapes of the LSF are complex, care must be taken when applying these three approaches to other cases; the reason is that several local minima, with similar values of their associated reliability indexes, may exist, so that the probability of failure might be underestimated. The collocation-based stochastic response surface methodology (CSRSM) (Mollon et al. 2011a), which provides more reliability in this matter, because it covers the whole parametric space and is able to detect local minima, appears as an interesting option in this case.] To assess the influence of the face support pressure on the reliability results, face support pressures are varied from 15 to 40 kPa. Results are plotted in Fig. 6, which shows that reliability indexes computed by the three approaches are very similar; hence, the RSM approximates well to the LSF in the region with maximum probability of failure.

In relation to computational efficiency, to solve Eq. (9) with a tolerance of \( \Delta \beta < 0.001 \) normally requires four or five iterations with the RSM; whereas the LSF-FORM requires six or seven iterations, on average (although, depending on the chosen initial point, this number sometimes increases to more than 10). Additionally, because numerical derivatives need to be computed in FORM, and the real LSF needs to be sampled in RSM, the deterministic model needs to be evaluated seven times (with both LSF-FORM and RSM) in each iteration. (Approximately 15 min are required for running the deterministic model once on a PC with an Intel Core i3-2100 CPU with 3.10 GHz and 8 GB of RAM.) On the other hand, the computational costs of the RSM-based FORM and IS are negligible compared to the design point search algorithms that address the real LSF. Therefore, it is clear that the LSF-FORM is more computationally expensive than the RSM-IS and the RSM-FORM. Together with the observation in Fig. 5, it can be concluded that the RSM-IS is a rational approach, with less computational cost and proper accuracy, to address the reliability of a tunnel face stability problem. However, the LSF-FORM provides a better design point, which is useful in the sensitivity evaluation.

As expected, the reliability increases with an increasing support pressure. Using the results of RSM-IS as reference, for the case considered, the reliability index is 0.499 for a support pressure, \( \sigma_r \), of 15 kPa, which is too low to be acceptable for engineering design (U.S. Army Corps of Engineers 1997); whereas the reliability...
index increases sharply to 2.044 for \( \sigma_r = 25 \) kPa, and to 3.341— which is often considered acceptable in engineering practice—for \( \sigma_r = 40 \) kPa.

**Influence of Distribution Types and Correlation Structures**

Fig. 7 shows the reliability indexes computed using RSM-IS, for the aforementioned four cases of different distribution types and correlation structures. Minor differences attributable to correlation exist when variables are normally distributed, and the correlation structure has a weak effect on the reliability results in this case. When variables are nonnormal, however, the correlation structure is more important, with differences between uncorrelated and correlated variables increasing with increasing support pressure.

The type of distribution under consideration is also important, with significantly lower reliability indexes computed for normal than for nonnormal variables. This is attributable to differences between the left tails of normal and nonnormal distributions (e.g., the left tail of a normal distribution theoretically extends up to \(-\infty\), whereas for a lognormal distribution, for instance, it is bounded by 0). With increasing support pressures, the design point tends to move toward the left tail of resistance variables. Because all variables considered herein are resistance variables, differences in their left tails can enlarge the differences of the computed reliability indexes. These results emphasize the importance of the proper selection of distribution types for the random variables under consideration, which should become a primary focus of future research in similar reliability assessments for engineering practice. On the other hand, the distribution tails of the input variables are probably incorrect, so the associated reliability estimators are also probably wrong in the absolute sense, regardless the accuracy of the reliability method used to compute them (MCS, RSM, IS, or FORM), and they are therefore only useful as decision-making tools, to compare among several designs.

**Sensitivity Study**

The base case, with support pressures varying within a 15–40 kPa range, is applied to compute the sensitivity of reliability results to changes in random variables for cases with different distribution types and correlation structures. Table 3 presents the computed sensitivity results, \( \gamma \), together with the computed design points, \( x' \). As expected, the components of the sensitivity vectors are all negative, confirming that the three random variables are resistance variables. Additionally, the differences in sensitivity between normal and nonnormal variables are more relevant than those between uncorrelated and correlated variables.

To better understand the different sensitivities for different distribution types, taking the results with uncorrelated normal variables as an example, it can be observed that the importance of \( \sigma_{ci} \) generally increases with increasing support pressures, reaching \(-0.87\) when the support pressure is equal to 40 kPa. This means that, for higher support pressures, \( \sigma_{ci} \) is more relevant to reliability than GSI and \( m_i \). On the other hand, GSI is the most relevant variable for tunnel faces with low support pressure. For uncorrelated nonnormal variables, however, variables are more similarly relevant (in order of decreasing importance, they are \( \sigma_{ci} \), GSI, and \( m_i \)), and their sensitivities are almost independent of support pressure. Similar observations can be found with correlated variables.

**Influence of Correlation Structure**

A slight influence of the assumed correlation structures on the computed reliability indexes is shown in Fig. 7. However, the correlation coefficients in use are relatively small. Therefore, it may be questioned how reliability results would change for other values of the correlation coefficients.

**Table 3. Sensitivities Computed for Different Support Pressures**

<table>
<thead>
<tr>
<th>( \sigma_r ) (kPa)</th>
<th>( \sigma_{ci} ) (MPa)</th>
<th>GSI</th>
<th>( m_i )</th>
<th>( \sigma_{ci} ) (MPa)</th>
<th>GSI</th>
<th>( m_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>15</td>
<td>-0.59</td>
<td>2.8</td>
<td>-0.74</td>
<td>14.1</td>
<td>-0.31</td>
<td>5.9</td>
</tr>
<tr>
<td>20</td>
<td>-0.59</td>
<td>2.5</td>
<td>-0.73</td>
<td>12.3</td>
<td>-0.34</td>
<td>5.7</td>
</tr>
<tr>
<td>25</td>
<td>-0.65</td>
<td>2.1</td>
<td>-0.67</td>
<td>11.3</td>
<td>-0.36</td>
<td>5.5</td>
</tr>
<tr>
<td>30</td>
<td>-0.73</td>
<td>1.8</td>
<td>-0.62</td>
<td>10.7</td>
<td>-0.30</td>
<td>5.5</td>
</tr>
<tr>
<td>35</td>
<td>-0.77</td>
<td>1.5</td>
<td>-0.57</td>
<td>10.4</td>
<td>-0.28</td>
<td>5.4</td>
</tr>
<tr>
<td>40</td>
<td>-0.87</td>
<td>1.1</td>
<td>-0.44</td>
<td>11.1</td>
<td>-0.23</td>
<td>5.5</td>
</tr>
</tbody>
</table>

The base case, with support pressures varying within a 15–40 kPa range, is applied to compute the sensitivity of reliability results to changes in random variables for cases with different distribution types and correlation structures. Table 3 presents the computed sensitivity results, \( \gamma \), together with the computed design points, \( x' \). As expected, the components of the sensitivity vectors are all negative, confirming that the three random variables are resistance variables. Additionally, the differences in sensitivity between normal and nonnormal variables are more relevant than those between uncorrelated and correlated variables.

To better understand the different sensitivities for different distribution types, taking the results with uncorrelated normal variables as an example, it can be observed that the importance of \( \sigma_{ci} \) generally increases with increasing support pressures, reaching \(-0.87\) when the support pressure is equal to 40 kPa. This means that, for higher support pressures, \( \sigma_{ci} \) is more relevant to reliability than GSI and \( m_i \). On the other hand, GSI is the most relevant variable for tunnel faces with low support pressure. For uncorrelated nonnormal variables, however, variables are more similarly relevant (in order of decreasing importance, they are \( \sigma_{ci} \), GSI, and \( m_i \)), and their sensitivities are almost independent of support pressure. Similar observations can be found with correlated variables.

**Influence of Correlation Structure**

A slight influence of the assumed correlation structures on the computed reliability indexes is shown in Fig. 7. However, the correlation coefficients in use are relatively small. Therefore, it may be questioned how reliability results would change for other values of the correlation coefficients.

<table>
<thead>
<tr>
<th>( \sigma_r ) (kPa)</th>
<th>( \sigma_{ci} ) (MPa)</th>
<th>GSI</th>
<th>( m_i )</th>
<th>( \sigma_{ci} ) (MPa)</th>
<th>GSI</th>
<th>( m_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
<td>( x' )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>15</td>
<td>-0.72</td>
<td>2.7</td>
<td>-0.57</td>
<td>14.6</td>
<td>-0.41</td>
<td>5.8</td>
</tr>
<tr>
<td>20</td>
<td>-0.69</td>
<td>2.3</td>
<td>-0.58</td>
<td>13.4</td>
<td>-0.44</td>
<td>5.5</td>
</tr>
<tr>
<td>25</td>
<td>-0.68</td>
<td>2.0</td>
<td>-0.60</td>
<td>12.4</td>
<td>-0.43</td>
<td>5.3</td>
</tr>
<tr>
<td>30</td>
<td>-0.67</td>
<td>1.8</td>
<td>-0.59</td>
<td>11.6</td>
<td>-0.45</td>
<td>5.0</td>
</tr>
<tr>
<td>35</td>
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<td>1.7</td>
<td>-0.63</td>
<td>10.7</td>
<td>-0.44</td>
<td>4.9</td>
</tr>
<tr>
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<td>-0.66</td>
<td>1.5</td>
<td>-0.60</td>
<td>10.3</td>
<td>-0.46</td>
<td>4.7</td>
</tr>
</tbody>
</table>
Table 4. Reliability Indexes for Different Correlation Coefficients

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation coefficient, ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>σci, GSI</td>
<td>-0.99 5.30 4.11 3.48 3.08</td>
</tr>
<tr>
<td>σci, mji</td>
<td>6.42 4.89 4.11 3.61 3.26</td>
</tr>
<tr>
<td>GSI, mji</td>
<td>6.02 4.81 4.11 3.64 3.32</td>
</tr>
</tbody>
</table>

Note: Case with nonnormal variables and 35 kPa support pressure. Each cell indicates the reliability index computed with the corresponding correlation coefficient between the variables, assuming that the correlations among others are 0.

Fig. 8. Reliability index as a function of tunnel diameter

The influence of the three correlation coefficients between σci, GSI, and mji is analyzed individually, comparing how these coefficients change the reliability results in relation to the base case with a support pressure of 35 kPa and nonnormal variables. Each cell in Table 4 indicates the reliability index computed with the indicated correlation coefficient between the variables, assuming that the correlations among others are 0. (To illustrate its relevance, the full range of possible correlation coefficients is considered herein; however, in reality, the absolute values of the correlation coefficients of rock mass variables might be smaller, which should be further investigated for each case.) Results show that these three correlation coefficients have a very similar, and relatively large, influence on the reliability index, in which large negative correlations lead to larger reliability indexes, and vice versa. These results suggest that, in practical engineering analysis and design, an effort should be made to estimate the actual correlation structure among random variables, so that, if possible, it should be obtained from actual rock mass data.

Parametric Study on Tunnel Diameter

To analyze the influence of diameter, a parametric study is conducted in which the diameter of the circular tunnel is changed from 4 to 14 m, for the base case of nonnormal correlated random variables. The support pressure is set to be 35 kPa in all cases.

Results in Fig. 8 show that the reliability index decreases rapidly with an increase of tunnel diameter. Small diameters (4 m) result in an extremely high reliability index (12.81), whereas large diameters (14 m) result in a very low reliability index (1.67). This confirms that, as is well known in practice, tunnels with larger diameter face higher construction risks associated with face collapses.

Conclusions

A reliability analysis is performed on a circular tunnel driven by a pressurized shield in an HB rock mass. The rock mass property parameters are considered as random variables with different distribution types and correlation structures. A limit analysis approach with an advanced rotational failure mechanism is used to assess stability. The LSF-FORM, RSM-IS, and RSM-FORM are applied to analyze the reliability against a collapse failure mechanism of the tunnel face. Different types of statistical distributions of the random variables involved, in addition to different correlation structures, are considered to analyze their influences on the computed reliability results. Similarly, sensitivities are computed to analyze how the reliability results are affected by changes to the input random variables, and sensitivity analyses are conducted to study the influences of changes of tunnel diameter and tunnel face pressure.

Results indicate that the reliability method employed—LSF-FORM, RSM-IS, or RSM-FORM—has a limited influence on computed results, hence suggesting that the LSF is not highly nonlinear. Also, the high computational cost associated with the LSF considered suggests the interest of using a computationally efficient alternative such as the RSM-IS (which is used in most of the analyses presented herein).

Sensitivity results show that all random variables under consideration—the uniaxial compressive strength of intact rock, σci, the rock mass quality given by GSI, and the mji parameter of the HB failure criterion—are resistance variables; therefore, a decrease in their mean values will reduce the reliability of the design, which will be primarily controlled by the left tails of their distributions. The sensitivity analyses also confirm the importance of tunnel diameter and tunnel face pressure on the reliability of the designs. In particular, the reliability results confirm that, as is well known in practice, the reliability of the tunnel face against collapse failure mode increases significantly (i.e., the probability of failure decreases) as the face support pressure increases or the tunnel diameter decreases.

Results also show that the type of random distribution (e.g., normal versus nonnormal distributions) has a significant influence of the computed reliability results. This observation, which agrees with previous observations about the importance of distribution types in the geotechnical literature (Jimenez and Sitar 2009), is primarily attributable to differences at the left tails of distributions. It also emphasizes the importance of an adequate characterization of geotechnical uncertainties for practical applications.

Similarly, the correlation structure, and the specific values of correlations, are found to be important (especially for the case of nonnormal variables), which emphasizes the importance of using adequate values of correlation coefficients—if possible, supported by real data—in practical application. In this sense, the observation in this study that a negative correlation exists between σci and mji (which is new in the literature) is likely to be useful to future researchers examining similar problems in rock mechanics.

Acknowledgments

The first author is supported by China Scholarship Council (CSC) and, for insurance coverage, by Fundación José Entrecanales Ibarra. The second author holds a Ph.D. fellowship from Fundación José Entrecanales Ibarra. This research is funded, in part, by the Spanish Ministry of Economy and Competitiveness, under grant BIA2012-34326. Dr. Jiayi Shen kindly provided information about
properties of different rock types. Their support is greatly appreciated. The authors also thank the reviewers for their valuable comments and suggestions.

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