Analysis and experimental study on thermal dispersion effect of small scale saturated porous aquifer

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ABSTRACT

The coefficient model of small scale thermal mechanical dispersion of saturated porous aquifer is established and it is applied in the heat transfer process of convective dispersion. Step-by-step test is conducted for the two physical processes of one-dimensional unsteady heat conduction of semi-infinite medium and convection dispersion to obtain heat physical parameters, thus achieving the verification purpose of analytical solution. On this basis, the porous aquifer thermal dispersion effect is evaluated, the results show that if coefficient of thermal mechanical dispersion $1 \times 10^{-2}$ W m$^{-1}$ K$^{-1}$ is selected as the critical point where thermal transport is affected, the distribution of thermal mechanical dispersion coefficients can be divided into non-ignorable triangular domain and ignorable polygon domain. However, the result shows that the maximum of longitudinal dispersivity is at centimeter order of magnitude, which is significantly different from the research result of thermal dispersivity under outdoors large scale conditions. This proves the existence of scale effect of thermal dispersion, and thus shows the direction of further research. At last, under condition that the thermal dispersion is ignored, the heat transfer method of thermal transport under conditions of different seepage velocities is also defined.

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1. Introduction

Thermal dispersion is a special heat transfer phenomenon in the thermal transport of porous aquifer. Due to the velocity fluctuation in the pore space of the medium, the heat is equalized and the heat transfer effect is strengthened. Therefore, the thermal transport and flow in porous media present many particular complex features, namely, the dispersion effects [1].

The early governing equation of convection conduction did not include thermal dispersion term, but with the gradual development of basic theory of heat transfer in saturated porous media, on the analogy of solute transport hydrodynamic dispersion effect, the discussion of conceptual model gradually focuses on thermal mechanical dispersion mechanism. Kwong et al. [2] proposed the concept of “effective thermal conductivity” and defined it as the sum of “stagnant liquid heat conductivity coefficient” and “thermal dispersion heat conductivity coefficient after the flow”. D Marsily [3] believed that thermal dispersion was similar to mechanical dispersion of the solute, the process was closely related to the structure of the medium and flow velocity. However, there is essential difference between thermal transport and solute transport, since the existence of heat conduction enables energy transfer among solid particles in the process of thermal transport and there are other thermal transports in the molecular diffusion and absorption effect in solute transport. It is thus clear that solute transport can be used as a reference but cannot be copied. A large amount of work has been done by many scholars for pore scale conditions, the thermal mechanical dispersion coefficient in the convective dispersion heat transfer process is generally regarded as a physical quantity that relies on the fluid velocity and particle size of porous media [4–8]. Metzger et al. [9] imitated porous media aquifer with a glass ball with a diameter of 2 mm and acquired relevant expressions of thermal-dynamic dispersion coefficient. Based on this research, N. Molina [10] discussed the influence of thermal dispersion effect on the thermal plume range in the geothermal system under microscale and macroscale conditions using analytical solution. Some scholars believe that thermal dispersivity is a function of size and cases show that in the energy storage experiment of large scale porous aquifer, the simulative temperature field data fit well with the field experiment data only...
on the premise that the thermal dispersion term is considered, there are defects for considering only the heat transfer method of convective conduction [11]. However, the dispute on convective dispersion of thermal transport in saturated porous aquifer has always existed. On the one hand, some scholars [12,13] believe that thermal dispersion \( \lambda_2/(\rho c)_a \) dominates in the simulative process of thermal transport and thermal mechanical dispersion is negligible relatively, on the other hand, many researches show that thermal dispersion effect is produced in micro-scale homogeneous porous media in the thermal transport process [1,14], which is of course also produced in the macro-scale heterogeneous thermal transport process [15–17], thermal dispersion effect is generally not negligible, especially under outside large scale conditions, it is generally believed that the longitudinal dispersivity is 5%–10% or one tenth of the radius of the influence [18]. In the simulative experiment of aquifer energy storage, \( \lambda_2/(\rho c)_a = 2 \) was applied [19]. Xue et al. [11] provided the longitudinal dispersivity as \( \alpha_r = 3.3 \) m in the seasonal energy storage experiment in Shanghai. However, as for the disputes on heat dispersion effect, we should take into consideration heat dispersion effect while doing calculation. From the perspective of development situation disclosed by dispersion mechanism, it is not mature yet. The bottleneck is lacking a unified calculation expression for heat mechanical dispersion coefficient. Moreover, these adjustable empirical constants are always different due to different experiment conditions and numerical simulations.

For example, Koch et al. [20] proposed a theoretical semiempirical correlation about axial and radial thermal dispersion effect, and some scholars tried to study the dispersion effect and turbulence model in porous media through direct simulation of the fluid flow and heat transfer in space with specific geometric shapes [21,22]. Chou et al. [23] compared some of the calculation methods and found out that the deviation was up to dozens of times. Most of the empirical coefficients were obtained through their own experiment data and the models lacked generality and comparability. At present, the researches of thermal mechanical dispersion coefficient mostly rely on direct measurements in experiments and numerical simulations. Yu [24] once provided some comprehensive analysis of the calculation formula of dispersion coefficient. There are various theoretical basis for the research on thermal dispersion: fractal analytical approach [25], turbulent mixing length theory [26,27], and numerical simulation of fluid and heat transfer in porous media [28,29], volume average method [30,31] and statistical average method [32].

At present, exploration is still constantly made for the study on heat dispersion effect. More theory and test research are necessary for its development. The dissertation employs small-scale physical model to study the influence of heat dispersion effect on heat transfer through test and analysis, thus laying a foundation for subsequent further study. In addition, accurate mastering of heat dispersion effect possesses great significance for accurate prediction of temperature field for the aquifer energy storage [33,34], GSHPs (Ground Source Heat Pumps) [35–37], nuclear engineering development technology, temperature field tracer, tunneling and large hydropower construction projects. It will also offer reference for study on more complex heat transfer of liquid—solid-heat coupling.

### 2. Experiments

#### 2.1. Experiment ideas

The experimental program is divided into two phases: (a) one-dimensional unsteady state heat conduction experiment of semi-infinite porous aquifer under non-convection condition, in this experiment, the heat is transported simply by heat conduction, by this physical process the thermal diffusivity \( \alpha_r \) of saturated porous aquifer is obtained, this is an indispensable parameter which cannot be obtained by instrument measurement in the study of thermal dispersion. (b) One-dimensional unsteady state convective dispersion experiment of semi-infinite porous aquifer under fixed

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( \alpha_l )</td>
<td>longitudinal dispersivity (m)</td>
</tr>
<tr>
<td>( \alpha_t )</td>
<td>transverse dispersivity (m)</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>thermal diffusivity of heat conduction process (m² s⁻¹)</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>effective thermal conductivity in the longitudinal direction (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>effective thermal conductivity in the transverse direction (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>thermal conductivity of water (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \lambda_4 )</td>
<td>thermal conductivity of the solid particles (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \lambda_5 )</td>
<td>empirical dispersivity coefficient (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \lambda_6 )</td>
<td>bulk thermal conductivity of porous medium (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>total thermal diffusivity (m² s⁻¹)</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>thermal diffusivity of heat conduction process (m² s⁻¹)</td>
</tr>
<tr>
<td>( \alpha_t )</td>
<td>thermal diffusivity of water (m² s⁻¹)</td>
</tr>
<tr>
<td>( S )</td>
<td>dimensionless temperature standard deviation (dimensionless)</td>
</tr>
<tr>
<td>( c_f )</td>
<td>heat capacity of water (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( c_s )</td>
<td>heat capacity of solid particles (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( A )</td>
<td>longitudinal dispersivity empirical constants (dimensionless)</td>
</tr>
<tr>
<td>( H )</td>
<td>aquifer thickness (m)</td>
</tr>
<tr>
<td>( \Delta H )</td>
<td>head difference (m)</td>
</tr>
<tr>
<td>( D )</td>
<td>specimen diameter (m)</td>
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<tr>
<td>( d )</td>
<td>particle diameter (m)</td>
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<tr>
<td>( \tau )</td>
<td>time (m s⁻¹)</td>
</tr>
<tr>
<td>( n )</td>
<td>total porosity (dimensionless)</td>
</tr>
<tr>
<td>( v )</td>
<td>Darcy velocity (m s⁻¹)</td>
</tr>
<tr>
<td>( \Delta z )</td>
<td>space step (m)</td>
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<tr>
<td>( \Delta \tau )</td>
<td>time step (s)</td>
</tr>
<tr>
<td>( \text{erfc} )</td>
<td>error function</td>
</tr>
<tr>
<td>( \theta )</td>
<td>dimensionless temperature (dimensionless)</td>
</tr>
<tr>
<td>( K )</td>
<td>hydraulic conductivity (m s⁻¹)</td>
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<tr>
<td>( T_0 )</td>
<td>initial temperature (K)</td>
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<tr>
<td>( T_s )</td>
<td>surface source temperature (K)</td>
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<tr>
<td>( T_{me} )</td>
<td>measuring temperature (K)</td>
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<tr>
<td>( q_c )</td>
<td>discharge per unit width (ml h⁻¹)</td>
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<tr>
<td>( Pe )</td>
<td>Peclet number (dimensionless)</td>
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<tr>
<td>( m_1 )</td>
<td>empirical constant (dimensionless)</td>
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<tr>
<td>( R_a )</td>
<td>assumed coefficient (dimensionless)</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>water density (kg m⁻³)</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>solid particle density (kg m⁻³)</td>
</tr>
<tr>
<td>( \rho_d )</td>
<td>dry density (kg m⁻³)</td>
</tr>
<tr>
<td>( w )</td>
<td>seepage velocity of test (m s⁻¹)</td>
</tr>
<tr>
<td>( (\rho c)_a )</td>
<td>porous medium heat capacity (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \rho c_{\text{fs}} )</td>
<td>heat capacity of water (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \rho s_{\text{fs}} )</td>
<td>heat capacity of solid skeleton (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>specific gravity (g/cm³)</td>
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</table>
drawdown condition, the purpose is to provide experimental verification for the establishment of small scale thermal mechanical dispersion model. The thermal transfer models in the two phases of experiment are shown in Fig. 1.

2.2. Experiment apparatus and parameters

The experiment apparatus is composed of high-precision constant temperature control system (model: XMTG-7000), temperature automatic acquisition system (model: SHWD-T485), temperature sensor, thermocouple, thin metal net, water level control device and one-dimensional earth-pillar model. The model is 0.8 m high, with a radius of 0.1 m, the aquifer is 0.3 m thick and the gravel layer at the bottom is 0.05 m, the thin metal net is on the top surface of aquifer, the wall surface of earth-pillar is thermal insulating material. See Fig. 2 for the experiment apparatus diagram.

The thermocouples were located to the places 0.08 m, 0.18 m and 0.28 m below the top surface of aquifer. In the experiment, the interface temperature was controlled accurately through the constant temperature control system, so as to satisfy the requirements of constant temperature boundary. The thermocouples were connected with the computer through temperature automatic acquisition system, data were acquired automatically with an acquisition accuracy of (0.0625 K) and a minimum acquisition frequency of 1 s. The circular water level control system guaranteed a stable water flow velocity in the aquifer, the flows at the inlet and outlet were recorded by flow meter with an accuracy of (0.01 ml). The parameters involved in the experiment include: liquid phase, solid phase, pore media and parameters of experiment apparatus. The parameter values are shown in Table 1.

2.3. Thermal physical parameter inversion

Before measuring the physical parameters in saturated porous media aquifer, purified water was used to check the experiment system. The thermal diffusivity of purified water measured had an error of less than 4% related to the reference value, which indicated that the apparatus can be used to measure the thermal diffusivity and other physical parameters of saturated porous media.

According to the mathematical model of (a) the one-dimensional unsteady heat conduction in semi-infinite aquifer, the unsteady heat conduction analytical solution is [38]:

\[
\theta(z, \tau) = \text{erfc} \left( \frac{z}{2 \sqrt{\alpha c \tau}} \right) = 1 - \text{erf} \left( \frac{z}{2 \sqrt{\alpha c \tau}} \right)
\]

(1)

where, \( \theta = (T_{z, \tau} - T_0)/(T_1 - T_0) \) is the dimensionless temperature, \( T_{z, \tau} \) is the temperature of any point at different moments in the aquifer, \( T_0 \) is the initial temperature in the aquifer, \( T_1 \) is the temperature at the upper boundary, \( \alpha c \) is the thermal diffusivity under heat conduction condition, \( \tau \) is the time, \( \text{erf}(\eta) \) is the error function, and \( \text{erfc}(\eta) \) is the complementary error function.

Since the thickness of the saturated porous aquifer in the experiment is limited, while the calculation model is established on the assumption of semi-infinite aquifer, therefore, it is necessary to analyze the reliability of experiment results. Dimensionless temperature analytical solution contains the dimensionless parameter \( u = z/2\sqrt{\alpha c \tau} \) and the complementary error function \( \text{erfc}(u) \). From the change rule of complementary error function \( \text{erfc}(u) \) with \( u \) it can be seen that: when \( u = 1.8 \), \( \theta(z, \tau) = \text{erfc}(u) = 0.0109 \), when \( u \geq 1.8 \), namely, \( z/2\sqrt{\alpha c \tau} \geq 1.8 \), the temperature at position \( z \) can be regarded as the initial temperature \( T_0 \), at this time, the dimensionless temperature error is about 1%, which can satisfy the requirements of the experiment.

To get the thermal diffusivity \( \alpha c \) of saturated porous aquifer in the earth-pillar, analytical fitting method is applied. Let \( f(z, \tau) = \text{arcercf} \left( \theta(z, \tau) \right), \tau' = 1/\tau \), and we get:

\[
f(z, \tau') = \frac{z}{2 \sqrt{\alpha c \tau'}}
\]

(2)

Two measuring points \( z_1 = 0.08 \) m and \( z_2 = 0.18 \) m underneath the upper boundary of the aquifer are selected for the calculation. Standard curves of four sets of analytical solutions and the correlation curves between two sets of experiment results and analytical solutions are given in Fig. 3.

The actual measured points of \( f(z, \tau) \) are drawn according to experiment results, and by method of least squares fitting, the
fitted curvilinear equation obtained is: \( f(0.08, \tau) = 72.5276\tau^2 + 0.001; f(0.18, \tau) = 164.8057\tau^2 - 0.0005 \)

Through the straight slope \( k \), we calculate the thermal diffusivity \( a_c \), and the volumetric heat capacity \( (\rho c)_f \) of the porous media aquifer can be expressed as [16]:

\[
(\rho c)_f = \frac{n f c_f + (1 - n) f c_s}{a_c}, \quad (3)
\]

\[
(\rho c)_f = \frac{3.082 \times 10^{-7} m^2 s^{-1}}{0.08}, \quad a_c = \frac{2.942 \times 10^{-7} m^2 s^{-1}}{0.18}, \quad a_c = \frac{3.012 \times 10^{-7} m^2 s^{-1}}{0.08}.
\]

Based on \( \lambda_s = a_c (\rho c)_f \), we can obtain the porous media stagnant heat conductivity coefficient \( \lambda_s \).

Based on the size of existing physical model, in terms of experiment time, if the control time \( \tau \leq 449 \) min is reliable, and it conforms to theoretical requirements to conduct parameter fitting with the time taking a value as 440 min when conducting thermal diffusivity parameter inversion.

### 3. Establishment of thermal mechanical dispersion

The effects of groundwater flow on temperature field have to be considered when studying the thermal transport involved in the energy storage of aquifer and in the utilization process of GSHPs shallow geothermal energy. The thermodynamic dispersion coefficient in convective dispersion model is composed of stagnant heat conductivity coefficient \( \lambda_s \) and thermal mechanical dispersion coefficient \( \lambda_v \), the longitudinal thermodynamic dispersion coefficient can be expressed as:

\[
\lambda_k = \lambda_s + \lambda_v \quad (4)
\]

Stagnant heat conductivity coefficient \( \lambda_s \) refers to the heat conduction coefficient in the aquifer when the groundwater is stagnant. The common calculation expression is adopted [39]. While in the paper, it is obtained by experiment, since the skeleton heat conductivity coefficient is difficult to determine.

\[
\lambda_s = (\rho c)_f \left[ \frac{\lambda_f}{\rho_f c_f} + (1 - n) \frac{\lambda_s}{\rho_s c_s} \right] \quad (5)
\]

where, \( (\rho c)_f \) is heat capacity of aquifer, \( n \) is the porosity of the aquifer, \( \lambda_f \) and \( \lambda_s \) are respectively the heat conductivity coefficients of water and solid skeleton in porous media, \( c_f \) and \( c_s \) are respectively the specific heat capacity of water and solid skeleton in porous media.

According to related expressions of thermodynamic dispersion coefficient provided by Metzger et al. [9]:

\[
\lambda_k = \lambda_s + A_P e^{m_1} \quad (6)
\]

\[
\lambda_k = \lambda_s + A_{min/max} e^{m_1} \quad (7)
\]

\[
Pe = \frac{\rho_f c_f d}{\lambda_f} \quad (8)
\]

where, \( Pe \) is the Peclet number, representing the relative proportion between convection and diffusion energy, \( d \) is the average particle size. Here, when calculating the longitudinal thermodynamic dispersion coefficient \( \lambda_k \), \( A = 0.073 \) and \( m_1 = 1.59 \), when calculating the horizontal thermodynamic dispersion coefficient \( \lambda_k \), \( A_{min} = 0.03 \), \( A_{max} = 0.05 \) and \( m_1 = 1 \), it is established on condition of homogeneous and isotropic media. The parameters dependency of this model is widely accepted in engineering application and some scholars apply it for the development and research of shallow geothermal energy. Another widely concerned thermal mechanical dispersion coefficient model is proposed by Sauty et al. [40] and Marsily [16] et al., which is similar to solute transport, in this model, the thermal mechanical dispersion coefficient is related to flow velocity, and in the study of energy storage of porous media aquifer, scholars believe that \( \lambda_v \) is in proportion to \( |v|^m \) \((m = 1, 2)\), where \( m = 1 \) is more widely used. The relational expression of thermal mechanical dispersion coefficient is shown by formula [9], which is relatively widely used for heterogeneous aquifer or if the aquifer structure is unknown [18,41].

\[
\lambda_v = a_k |v| \quad (9)
\]

By substituting formula (9) into formula (4), we get:

\[
\lambda_k = \lambda_s + a_k |v| \quad (10)
\]
\[ \lambda_y = \lambda_a + a_y \rho_f c_f |v| \]  
(11)

where, \( \lambda_a \) and \( a_y \) are respectively the longitudinal and horizontal thermal dispersivity. In the experiment, the horizontal thermal dispersivity is ignored, the flow velocity in the model satisfies \(|w| = |v|\), by substituting formula (10) into formula (6) and combining formula (8) we can get the longitudinal dispersivity expression as follows:

\[ a_x = A \left( \frac{\rho_f c_f |w|}{\lambda_t} \right)^{m-1} \]  
(12)

By substituting formula (12) into formula (9), we can deduce the expression of thermal mechanical dispersion coefficient in the experiment as follows:

\[ \lambda_y = a_x \rho_f c_f |w| = \lambda_t A \left( \frac{\rho_f c_f |w|}{\lambda_t} \right)^{m-1} \]  
(13)

\[ \lambda_y = \lambda_a + \lambda_t A \left( \frac{\rho_f c_f |w|}{\lambda_t} \right)^{m-1} \]  
(15)

\[ a_x = \alpha_c + \lambda_t A \left( \frac{\rho_f c_f |w|}{\lambda_t} \right)^{m-1} \]  
(16)

**4. Comparison and verification**

**4.1. Numerical calculation**

According to the experiment conditions of semi-infinite earth-pillar model, assuming that the aquifer is homogenous and isotropic without any deformation, the medium skeleton and the groundwater are regarded as equivalent continuous medium since they have the same temperature. As shown in the Fig. 1(b), at the moment \( \tau = 0 \), the initial temperature of the aquifer is \( T_0 \), the temperature of the top surface boundary is \( T_s \), the seepage velocity is \( w \), then, one-dimensional convective dispersion heat transfer exists in the saturated porous aquifer. When the longitudinal thermodynamic dispersion coefficient \( \lambda_a \) is a constant and there is no internal heat source, when not considering the heat transfer in \( x \) and \( y \) directions, the unsteady state one-dimensional convective dispersion model in the experiment can be expressed as:

Let \( \theta = \frac{T_i - T_0}{T_s - T_0} \)

\[ \begin{cases} 
\left( \frac{1}{c_f} \right) \frac{\partial \theta}{\partial \tau} = \frac{\lambda_a}{\rho \lambda_t} \frac{\partial^2 \theta}{\partial z^2} & 0 < z < \infty, \tau > 0 \\
\theta(z, 0) = 0 & 0 < z < \infty, \tau = 0 \\
\theta(0, \tau) = 1 & z = 0, \tau > 0 \\
\theta(\infty, \tau) = 0 & z \to \infty, \tau > 0 
\end{cases} \]  
(14)

where, the longitudinal thermodynamic dispersion coefficient \( \lambda_a \) and the thermal diffusivity \( \alpha \) can be expressed as:

**Formula (14)** can be written in convective dispersion form similar to solute transport:

\[ \begin{cases} 
\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial z} \left[ \frac{\rho c_f}{\lambda_t} \frac{\partial \theta}{\partial z} \right] & 0 < z < \infty, \tau > 0 \\
\theta(z, 0) = 0 & 0 < z < \infty, \tau = 0 \\
\theta(0, \tau) = 1 & z = 0, \tau > 0 \\
\theta(\infty, \tau) = 0 & z \to \infty, \tau > 0 
\end{cases} \]  
(17)

Characteristic finite analysis form is applied in the model for the experiment:

The hydrodynamic force derivative is defined as:

\[ \frac{D \theta}{D \tau} = \frac{\partial \theta}{\partial z} + \frac{w \partial \theta}{R_d \partial z} \]  
(18)

By substituting it into the control equation in mathematical model (17), we obtain:

\[ \frac{D \theta}{D \tau} = \alpha \frac{\partial^2 \theta}{\partial z^2} \]  
(19)

The characteristic equation of formula (18) can be expressed as:

\[ \frac{dz}{d\tau} = \frac{w}{R_d} \]  
(20)

In local grid, we take \( dz/d\tau = \theta_i^{j+1} - \theta_i^j / \Delta \tau \), where, \( \Delta \tau \) is the time step, \( \theta_i^j \) is the dimensionless temperature value at site \( z_i \) at \( \tau \) moment, then, the formula can be written as:

\[ \theta_i^{j+1} - \theta_i^j = \Delta \tau \frac{\partial^2 \theta}{\partial z^2} \]  
(21)

Let \( f = R_d \theta_i^{j+1} - \theta_i^j / \Delta \tau \), by substituting \( f \) into equation (21), we get the general solution form of the ordinary differential equation:

\[ \theta_i = f \cdot \left( \frac{1}{2} z_i^2 + C_1 z_i + C_2 \right) \]  
(22)

To get the integral constant, we introduce the boundary conditions \( \theta_i^{j+1} \) and \( \theta_i^{j+1} \) of dimensionless temperature \( \theta \) in local units at moment \( \tau + \Delta \tau \), by substituting into the above formula, we get:

\[ \begin{align*}
C_1 &= \frac{\theta_i^{j+1} - \theta_i^{j+1}}{f(z_i + 1 - z_i - 1)} - \frac{1}{2} (z_i + 1 + z_i - 1) \\
C_2 &= \frac{-\theta_i^{j+1} + \theta_i^{j+1}}{f(z_i + 1 - z_i - 1)} + \frac{1}{2} z_i + z_i - 1
\end{align*} \]  
(23)

Then, by substituting \( C_1 \) and \( C_2 \) into formula (22), we get the dimensionless temperature value \( \theta \) at site \( i \) at moment \( \tau + \Delta \tau \), namely:

\[ \theta_i^{j+1} = f \cdot \left( \frac{1}{2} z_i^2 + C_1 z_i + C_2 \right) \]  
(24)

Then, regain the form of \( f \), we get the characteristic finite analysis form:

\[ f = \theta_i^{j+1} - \theta_i^{j+1} = \frac{\theta_i^{j+1} (z_i + 1 - z_i - 1) + \theta_i^{j+1} (z_i + 1 - z_i - 1)}{(z_i + 1 - z_i - 1)} \]  
(25)

Assuming \( R_d = \frac{(|v|)_s}{\rho_f c_f} \)}
If considering equal distance subdivision for the unit, assuming the step size is $\Delta x$, the above formula can be written as:

$$
\frac{\alpha \Delta \tau}{R_d \Delta x^2} \phi_i^{j+1} + \left( 1 + \frac{2 \alpha \Delta \tau}{R_d \Delta x^2} \right) \phi_i^j - \frac{\alpha \Delta \tau}{R_d \Delta x^2} \theta_{i+1}^j - \theta_i^j = 0
$$

(26)

It is called characteristics finite analysis form that the convective dispersion equation form established based on the method of characteristics and finite analytic method. The numerical solution method has been well verified in literature [42], in the paper, Matlab programming is used for the calculation and contrastive analysis with analytical solution.

### 4.2. Analytical solution

For mathematical model (14), through Laplace transformation, inverse transformation and convolution theorem we can see that the solution form acquired is similar to the analytical solution of convective transport [43], the solution form of one-dimensional convective dispersion thermal transfer in the paper is:

$$
\theta = \frac{T_2 z - T_0}{T_2 z_0 - T_0} = \frac{1}{2} \text{erfc} \left( \frac{z - w \frac{\partial C}{\partial t} \tau}{\sqrt{\frac{2 \alpha}{\partial x} \tau}} \right) + \frac{1}{2} \exp \left( \frac{-w \frac{\partial C}{\partial t} z}{\alpha + \frac{\partial C}{\partial x} \frac{\partial w}{\partial x} \tau} \right) \text{erfc} \left( \frac{z + w \frac{\partial C}{\partial t} \tau}{\sqrt{\frac{2 \alpha}{\partial x} \tau}} \right)
$$

(27)

The approximate representation of related complementary error function $\text{erfc}(\eta)$ is [44]:

$$
\text{erfc}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\eta}^{\infty} \exp(-y^2) dy
$$

$$
= \begin{cases} 
1 - \sqrt{1 - \exp(-c^2 \eta^2)}, & (\eta \geq 0) \\
1 + \sqrt{1 - \exp(-c^2 \eta^2)}, & (\eta < 0)
\end{cases}
$$

(28)

where $c < 2$, $c$ is a constant, to simplify the calculation, within the margin of error, a constant value is taken in interval $[-\infty, +\infty]$, and by method of optimization it is calculated. We know that $\text{erfc}(\eta)$ decreases with the increases of $\eta$, and if $\eta$ is large enough, namely that the direction is large enough or the $\tau$ is long enough, then, formula (28) can be calculated based on the property of normal distribution function.

### 4.3. Numerical value and experimental verification of analytical solution

A small scale thermal mechanical dispersion coefficient model is established under homogeneous and isotropic conditions and it is applied in the mathematical model of convective dispersion. Characteristic finite analysis form is applied for the solution. In addition, the analytical solution of mathematical model of convective dispersion heat transfer established under small scale conditions is deduced. At last, the analytical solution and the numerical solution are compared to verify the reliability of analytical solution, then the experiment result and the analytical solution are compared, so as to further discuss the correlation between convective dispersion heat transfer process of saturated porous aquifer and factors such as media characteristics, hydrogeological conditions and physical parameters. This lays foundation for the influence study of thermal dispersion effect on thermal transport.

According to mathematical model (14), characteristic finite analysis form is used for the solution, the time step is $\Delta \tau = 10$ min, the space step is $\Delta x = 0.02$ m, the simulated time is 5000 min. To verify analytical expression (27), two measuring points $z_1 = 0.08$ m and $z_2 = 0.18$ m are selected in the aquifer, the comparison results of the dimensionless temperature change with time acquired through calculation are shown in Fig. 4, see Table 1 for the parameter values in the calculation process.

To testify the fitting relationship between analytical solution and numerical solution, RMSE (root mean square error) is applied to calculate the errors of the two calculated results. A dimensionless data sample with the dimensionless temperatures at different moments is built and the standard deviation can be expressed as:

$$
S = \sqrt{\frac{\sum_{i=1}^{n} (\hat{\theta}_i - \bar{\theta})^2}{\delta}}
$$

(29)

where, $\hat{\theta}_i$ is the dimensionless temperature calculated by analytical solution, $\bar{\theta}$ is the dimensionless temperature calculated by numerical solution, $\delta$ is the calculation times, the calculation result of standard deviation is: RMSE ($z_1 = 0.08$ m) = 0.0045, RMSE ($z_2 = 0.18$ m) = 0.0077. It shows a good fitting effect, and thus proves that analytical solution can reflect the heat transfer process of convective dispersion. As an empirical threshold, consider that RMSE values <0.1 as standard to determine the effect of fitting. This is based on the typical measurement accuracy of temperature.

Under condition of the same parameters, the seepage velocity $w$ and thermal diffusivity $a_t$ measured in the one-dimensional earth-pillar are substituted into formula (27), assuming the longitudinal dispersivity $a_x$ takes values in the five orders of magnitudes such as: $10^{-2}$ m, $10^{-1}$ m, 0, $10^0$ m, $10^1$ m and $10^2$ m, the response curve of dimensionless temperatures of different longitudinal dispersivity with time is calculated through analytical solution, the experiment results of measuring point at site $z = 0.18$ m are drawn in Fig. 5.

It can be seen that under the condition of a flow velocity of $8.68 \times 10^{-7}$(m s$^{-1}$), if the longitudinal dispersivity $a_x$ takes value within the range of $[0, 10^{-2}]$ m, the change curve of dimensionless temperature with time acquired in the experiment will have a good correlation with the analytical solution.
fitting effect with the analytical solution, if the longitudinal dispersivity is larger than $10^{-1}$, the curve will deviate obviously. In the experiment, the longitudinal dispersivity is calculated to be $a_d = 5.53 \times 10^{-10}$ m based on the average particle diameters in aquifer, it is in the range of longitudinal dispersivity where the trial value has a good fitting with the analytical solution, that is to say, thermal dispersion effect has a relatively small influence on the dimensionless temperature under conditions in the experiment, which verifies the reasonability of the analytical solution in the paper. On this basis, it is necessary to discuss further the characteristics and energy storage in various aquifers and flow conditions in GSHPs technology engineering.

5. Evaluation of thermal dispersion effect of aquifer

To evaluate the influence of thermal mechanical dispersion effect on thermal transport of aquifer, it is discussed in aspects of hydrogeological conditions, physical parameters and aquifer media characteristics in the paper. Referring to literature [10], based on the assumption condition with a hydraulic gradient of $10^{-3}$, the reference values of typical hydraulic parameters and thermodynamic parameters in natural aquifer system are provided in Table 2. The variation range of hydraulic parameters is large, which exceeds 7 orders of magnitudes, in contrast, thermodynamic parameters have a low variability.

According to the reference values provided, the upper limit of Darcy velocity of aquifer under natural conditions is $1 \times 10^{-5}$ m s$^{-1}$, the lower limit is $1 \times 10^{-13}$ m s$^{-1}$, in addition, according to the particle fraction regulated by the Ministry of Water Resource of China [45]. The pebble group in macrosome group and the clay in fine grain group in aquifer are selected as the upper and lower limits for the calculation of particle diameters, the upper limit is 0.2 m, the lower limit is $3 \times 10^{-6}$ m.

<table>
<thead>
<tr>
<th>Aquifer material</th>
<th>Permeability coefficient $K$ (m s$^{-1}$)</th>
<th>Seepage velocity $w$ (m s$^{-1}$)</th>
<th>Thermal conductivity $\lambda_a$ (W m$^{-1}$ K$^{-1}$)</th>
<th>Medium heat capacity ($\rho c_p$) ($J$ K$^{-1}$ m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>$10^{-4}$–$10^{-2}$</td>
<td>$10^{-7}$–$10^{-5}$</td>
<td>1.5–2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>$10^{-3}$</td>
<td>$10^{-8}$</td>
<td>1.7–5.0</td>
<td>2.2–2.9</td>
</tr>
<tr>
<td>Medium sand</td>
<td>$10^{-4}$</td>
<td>$10^{-9}$</td>
<td>1.7–5.0</td>
<td>2.2–2.9</td>
</tr>
<tr>
<td>Fine sand</td>
<td>$10^{-6}$–$10^{-5}$</td>
<td>$10^{-9}$–$10^{-8}$</td>
<td>1.7–5.0</td>
<td>2.2–2.9</td>
</tr>
<tr>
<td>Silt</td>
<td>$10^{-7}$</td>
<td>$10^{-10}$</td>
<td>0.9–2.3</td>
<td>1.6–3.4</td>
</tr>
<tr>
<td>Clay</td>
<td>$10^{-10}$–$10^{-9}$</td>
<td>$10^{-11}$–$10^{-12}$</td>
<td>1–1.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>
the variation range and trend of longitudinal dispersivity of heterogeneous aquifers under natural seepage field are discussed. It is obvious that the longitudinal dispersivity in different particle diameter areas corresponding to different aquifers are divided into four areas by pebble layer, gravel layer, sand layer and clay layer, the longitudinal dispersivity increases with the increase of flow velocity and the relation in different aquifer structures is $a_{\text{pebble layer}} > a_{\text{gravel layer}} > a_{\text{sand layer}} > a_{\text{clay layer}}$.

The change characteristics are shown clearly through research on the change of longitudinal dispersivity in different aquifer structures, but its effect on thermal transport cannot be shown visually. The groundwater flow velocity range under natural flow condition has been given in the research of thermal transport of aquifer, but the seepage velocity rises greatly under local forced convection effect caused by pumping and pouring of groundwater involved in energy storage and GSHPs engineering. Therefore, $10^{-3}$ to $10^{-13}$ m s$^{-1}$ is selected to be the velocity variation range of further analysis.

According to analytical solution (27) and on the basis of calculating parameters $z = 0.5$ m and $\tau = 1000$ min, the thermal transport variation rules under different seepage velocities are discussed with consideration of longitudinal dispersivity ($a_x \neq 0$) and without consideration of longitudinal dispersivity ($a_x = 0$), dimensionless temperature difference $\Delta \theta$ is applied to represent the dimensionless temperature change produced when considering the thermal dispersion effect. The dimensionless temperature difference can be expressed as:

$$\Delta \theta = \theta_{a \neq 0} - \theta_{a = 0}$$

$\Delta \theta = \theta_{a \neq 0} - \theta_{a = 0} \leq 1 \times 10^{-3}$ is defined to be the standard that the temperature is not affected (namely, a $0.1^\circ$C change in $100^\circ$C is the standard), then from Fig. 8 it can be seen that it has no influence on the dimensionless temperature change in the thermal transport process whether to consider the thermal dispersion when the thermal mechanical dispersion coefficient $\lambda_m < 10^{-3}$ W m$^{-1}$ K$^{-1}$, when $\lambda_m > 10^{-2}$ W m$^{-1}$ K$^{-1}$, the consideration of thermal mechanical dispersion effect has influence on the thermal transport, the dimensionless temperature difference $\Delta \theta$ presents different variation rules with the variation of thermal mechanical dispersion coefficient under condition of different seepage velocities.

In addition, based on the variation range of longitudinal dispersivity acquired in Fig. 6, the variation rule of thermal dispersion coefficient under different flow velocities is discussed. Fig. 9 shows the change curve of thermal mechanical dispersion coefficient with the longitudinal dispersivity under different flow velocities, combining the analysis result in Fig. 8, it indicates that coefficient partition can be conducted for the thermal mechanical dispersion coefficient with longitudinal dispersivity under condition of different seepage velocities, according to whether thermal transport is influenced by the thermal dispersion effect under different aquifer structures.

From the analysis results in Fig. 8 it can be seen that, if the thermal mechanical dispersion coefficient value is selected as the critical point, the intersection domain of longitudinal dispersivity, thermal mechanical dispersion coefficient and groundwater seepage velocity can be divided into three domains, the upper triangular domain is
non-ignorable domain of thermal mechanical dispersion coefficient, the lower polygon domain is the ignorable domain, specific correlation is shown in Fig. 9. It is certain that when the longitudinal dispersivity \( \alpha_z < 10^{-6} \text{ m} \) and flow velocity \( w < 10^{-4} \text{ m s}^{-1} \), the thermal mechanical dispersion coefficient can be ignored in the calculation of thermal transport, when the longitudinal dispersivity \( 10^{-3} < \alpha_z < 6.9 \times 10^{-2} \text{ m} \) and flow velocity \( w > 10^{-3} \text{ m s}^{-1} \), it cannot be ignored, when the interval of longitudinal dispersivity is \([10^{-6}, 10^{-3}] \text{ m}\) and the flow interval is \([10^{-8}, 10^{-5}] \text{ m s}^{-1}\), it depends on specific conditions whether it can be ignored.

The evaluation of thermal dispersion effect in the process of thermal transport of aquifer under small scale condition shows that the convective dispersion result based on small scale thermal mechanical dispersion coefficient model is in conformity with the experiment result. The thermal dispersion effect relies only on the aquifer particle size and the water permeability, but the value range of thermal dispersivity is far from the research result of large scale convective dispersion. Attention shall be paid to the scale effect of thermal dispersion.

6. Heat transfer under different flow conditions

Based on the above analysis results, the heat conduction process of thermal transport can be divided into two sections when ignoring thermal mechanical dispersion in the experiment model, one is heat conduction effect and the other is convective heat transfer effect. However, in the case that other conditions remain unchanged, the seepage velocity determines the contribution of the two sections to heat transfer. Since the temperature change of measuring point can represent the amount of thermal transport, based on the verification of analytical solution in experiments (a) and (b), we calculate the dimensionless temperature under the effects of only the heat conduction and convective heat transfer, the difference value between the two represents the change of temperature field by convective heat transfer and reflects the dynamic relationship of convection and heat conduction in the thermal transport process.

As shown in Fig. 10(a)–(d), after remaining other conditions unchanged and changing seepage velocity only, the variation process of dimensionless temperature under heat conduction effect without seepage is in interval \([1 \times 10^{-8}, 5 \times 10^{-4}] \text{ m s}^{-1}\) with \(w = 0\). It can be seen that compared with under effect of convective heat transfer, the rising velocity of dimensionless temperature with time under heat conduction effect is obviously slow, with the increase of velocity, the contribution of convective heat transfer increases gradually, with the decrease of velocity, the contribution of heat conduction increases gradually, when the velocity is smaller than a certain value, the change curve of dimensionless temperature with time under effects of heat conduction and convection heat transfer nearly overlap, the contribution of convection heat transfer to thermal transport process varies constantly with the change of measuring points, therefore, with the temperature change of singly one certain measuring point to represent the contribution of convection and heat conduction, it cannot reflect the thermal transport process of the whole system. To reflect the ratio of convection heat transfer in the total heat transport with definite quantity, the total heat of the system is used to define.

According to Fourier's law, the heat flux density of heat conduction at any interface indirect of \(z\) can be expressed as:

\[
q_z = -\lambda \frac{\partial T}{\partial z}
\]  

(32)

Under the convection conduction effect, the heat flux density of heat conduction at any interface indirect of \(z\) can be expressed as:

\[
q_z = \rho c T \frac{\partial \theta}{\partial z} - \lambda \frac{\partial \theta}{\partial z}
\]

(33)

Then, the total heat conduction amount refers to the heat conduction amount from the upper boundary of the model to a designated infinite boundary, namely in the distance interval of \([0, z]\).

\[
Q_c = \int_0^z q_z \, dz
\]

(34)

Defining coefficient \(\varepsilon\) as the proportion of heat transfer caused by convection effect in the total heat transfer amount, then,

\[
\varepsilon = \frac{Q - Q_c}{Q}
\]

(36)

According to Fig. 11, we get the variation range of \(\varepsilon\), as shown in Table 3. From the table it can be seen that the variation range of \(\varepsilon\) is very large. When the seepage velocity \(w \geq 5 \times 10^{-5} \text{ m s}^{-1}\) and \(\varepsilon \geq 91.34\%\), the aquifer system is affected mainly by convection heat transfer, the heat conduction effect occupies no more than 9\% and convection heat transfer dominates in the system, when the seepage velocity \(5 \times 10^{-8} < w < 5 \times 10^{-3} \text{ m s}^{-1}\) and \(0.43\% < \varepsilon < 91.34\%\), the aquifer system heat transfer is affected by the combined effects of convection and heat conduction and both their effects shall be considered when calculating, when the seepage velocity \(w \leq 5 \times 10^{-8} \text{ m s}^{-1}\) and \(\varepsilon \leq 0.43\%\), the convection heat transfer occupies a small proportion, the convection effect can be ignored when it is lower than 1\%, that is to say, the thermal transport can be regarded as contributed totally by heat conduction effect for its calculation. The definition of the effect of seepage velocity on the thermal transport provides a reference for the aquifer energy storage and whether to consider the effect of natural seepage field on temperature field in GSHPs technology engineering.

7. Conclusions

Aiming at the influence study of saturated porous aquifer thermal dispersion effect on thermal transport, it puts forward a new test scheme during the heat dispersion effect researches, which, on the theoretical basis of heat mechanical dispersion coefficient model under the small-scale condition, deduces the analytical solution of heat convection and dispersion under this condition, and then evaluates heat dispersion effect of aquifer. Also, under condition that the thermal dispersion is ignored, the heat transfer method of thermal transport under conditions of different seepage velocities is also defined.

(1) We discussed whether to consider the effect of thermal mechanical dispersion coefficient on the thermal transport under conditions of different seepage velocities. Dimensionless temperature difference \(\Delta \theta\) was applied to represent the dimensionless temperature change when considering the thermal dispersion effect. Then we obtained that when the thermal mechanical dispersion coefficient \(\lambda_m < 10^{-2} \text{ W m}^{-1} \text{ K}^{-1}\), the thermal mechanical dispersion would not affect the thermal transport; when
The effect of thermal mechanical dispersion on the thermal transport was negligible; moreover, the dimensionless temperature difference presented different nonlinear change rules with the change of thermal mechanical dispersion coefficient under condition of different seepage velocities.

(2) The results show that if coefficient of thermal mechanical dispersion $\lambda_0 > 10^{-2}$ W m$^{-1}$ K$^{-1}$ is selected as the critical point where thermal transport is affected, through the evaluation of thermal dispersion effect, we achieved the partition of thermal mechanical dispersion coefficients which showed the effect of thermal dispersion on the

![Graphs showing dimensionless temperature change with time and distance under different seepage velocities.](image)

Table 3

<table>
<thead>
<tr>
<th>Seepage velocity (m s$^{-1}$)</th>
<th>$\varepsilon$ (%)</th>
<th>Seepage velocity (m s$^{-1}$)</th>
<th>$\varepsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{-5}$</td>
<td>91.34</td>
<td>$1 \times 10^{-7}$</td>
<td>423</td>
</tr>
<tr>
<td>$1 \times 10^{-5}$</td>
<td>88.47</td>
<td>$5 \times 10^{-8}$</td>
<td>2.14</td>
</tr>
<tr>
<td>$5 \times 10^{-6}$</td>
<td>77.82</td>
<td>$1 \times 10^{-8}$</td>
<td>0.43</td>
</tr>
<tr>
<td>$1 \times 10^{-6}$</td>
<td>33.85</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$5 \times 10^{-7}$</td>
<td>19.13</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 10. Change of dimensionless temperature with time under condition of different seepage velocities at different measuring points.

Fig. 11. Change curve of dimensionless temperature with distance under condition of different seepage velocities.
thermal transport under small scale condition. The distribution of thermal mechanical dispersion coefficients can be divided into non-ignorable triangular domain and ignorable polygon domain. The calibration scope of longitudinal dispersivity of natural flow field in different aquifer, however, seeing from the research result of small scale thermal dispersion, under condition that the particle diameter and the seepage velocity are both at the upper limit, the maximum of longitudinal thermal dispersion degree is at centimeter order \((6.9e \times 10^{-2} \text{ m})\) of magnitude. The longitudinal dispersivity in different particle diameter areas corresponding to different aquifers is divided into four areas by peddle layer, gravel layer, sand layer and lay layer.

(3) Under condition of ignoring the thermal dispersion effect, we defined the contribution of convective heat transfer in the total heat transfer with the total heat in the system, we also defined the effect of different seepage velocities on thermal transport. When the seepage velocity \(w \geq 5 \times 10^{-4} \text{ m s}^{-1}\), the convective heat transfer dominated in the system, when the seepage velocity \(5 \times 10^{-5} < w < 5 \times 10^{-4} \text{ m s}^{-1}\), both the two heat transfer methods affected, when the seepage velocity \(w \leq 5 \times 10^{-4} \text{ m s}^{-1}\), the thermal transport could be regarded as contributed totally by heat conduction effect for its calculation.

Seeing from the research results of small scale thermal dispersivity, there are great differences with the results of thermal dispersion effect under large scale condition. Therefore, it is the direction for follow-up research on thermal dispersion scale effect under outside large scale condition.

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