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Experimental study on the maximum impact force by rock fall

Abstract The maximum impact force caused by a rockfall is a very important factor for the design of protection measures for houses, roads, and bridges. To establish an impact force model, the kinetic energy of the rock block, the impact angle between the movement direction of the block and the surface of the object hit by the block, and the modulus of elasticity of rock and object were analyzed with the Buckingham theorem and simplified to two dimensionless parameters. Physical tests were conducted with different kinetic energies, moduli of elasticity of the rock, and the object. Balls of iron, granite, marble, sandstone, and wood were used to simulate rock blocks in the tests. The objects hit by the balls are composed of steel, concrete, and wood. The relationships between the maximum impact force and the kinetic energy, and modulus of elasticity determined by dimensional analysis were confirmed by these experiments. Experiments were carried out with different impact angles to determine the influence of the impact angle on the impact force. A maximum impact force model is obtained from these relationships and by experiments with impact forces ranging from 225 to 15,583 N. A comparison with results reported from other studies shows that the maximum impact force model gives reasonable results over a very large range of impact forces from 21.4 to 8.16 MN. We assume that the model can be used to calculate the impact force at the full field scale.

Keywords Impact force · Rock fall · Dimensional homogeneity · Modulus of elasticity

Introduction

Houses, roads, and bridges constructed in mountainous areas are exposed to the danger of rockfalls due to rainfall, weathering, and earthquakes. The estimation of maximum impact forces caused by a rockfall is an important factor for the design of protection measures for houses, roads, and bridges. Previous research in this field was carried out by Calvetti et al. (2005). They investigated experimentally and numerically the phenomenon of boulder maximum impacts on granular soils that are used to reduce loads on shelters. They performed four series of prototype scale tests by dropping a reinforced concrete boulder on soil strata with different geometrical and mechanical properties. The influence of block mass, falling height, and thickness of the protective cushion layer were analyzed numerically, and empirical correlations were established between impact energy, impact force, and stresses on the shelter below the protective cushion layer. They pointed out by their correlation between the results of experiments, and numerical simulations showed that the impact force on the shelter is linearly related to the impact energy of the boulder on the cushion layer.

Some researchers defined the maximum impact force in terms of the energy of the rock block, without using the modulus of elasticity of rocks and objects that were hit (Berthet-Rambaud et al. 2004; Delhomme et al. 2007; Dorren and Berger 2005; Mavrouli and Corominas 2010; Ronco et al. 2009). As the impact force is not only the result of block energy, but also the result of interaction of block and object, we assume that the Young's modulus of elasticity should be considered in the determination of the impact force.

Kawahara and Muro (2006) investigated the effects of the dry density and thickness of a sand cushion layer on an impact response due to a falling weight such as a rockfall. The Lame's constant of the sand cushion, which is the function of the elastic modulus and the Poisson ratio, was introduced in the equation to calculate the impact force. Pichler et al. (2005, 2006) also studied the behavior of layers of gravel as an energy-absorbing system for structures subjected to rockfall. The relations between the penetration depth, the impact duration, and the impact force on the one hand, and the rock boulder mass, the height of fall, and the indentation resistance of the gravel were investigated. Montani-Stoffel (1998) and Volkwein et al. (2011) showed that the impact force on the cushion layer is a function of the E-Moduli of the cushion layers, the block radius, and the rock's kinematic energy, expressed in terms of mass and impact velocity. These studies introduced the elastic modulus of the cushion layer in the calculation of the impact force, but without consideration of the elastic modulus of the block material, which will have an important effect on the impact force when there is no cushion layer.

Zhang et al. (1995), Hungr et al. (1984), and Timoshenko and Goodier (1970) calculated the impact force from the velocity of rock, the mass of rock, the modulus of elasticity of rock and object, the Poisson ratio of rock and object, and the radius of the rock block. Mavrouli et al. (2016) calculated also the impact force using the PFEM method and considered the rock-structure interaction and their properties. Some of the impact forces of some of the examples calculated by the equation of Zhang et al. (1995) are too large to be used in hazard prevention. Although these studies considered the modulus of elasticity of rock and object, the equations used are not dimensionally homogeneous. This causes problems of scale, especially for the extremely large scale in the field, as described by Zhang et al. (1995).

In this paper, we develop a calculation model for the maximum impact force of a rock block on an object with or without cushion layer, which is valid for all scales, using the Buckingham theorem. Laboratory experiments were carried out to determine the influence of the modulus of elasticity of the rock block material and of the hit object on the impact force. The results of these experiments may also be applied in the large in situ scale because of the dimensional homogeneity. This was partly proven by comparing the results of the small- and large-scale experiments with or without cushion layer reported by other authors.

The Buckingham theorem

The impact force by a rock fall on an object is determined by the change of energy with a short penetration distance (Eq. 1):

$$F = \frac{\Delta E_k}{l} \quad (1)$$

in which F is the impact force (N) and E_k is the kinetic energy of the rock block (J). l is the penetration distance by the rock (m).

The kinetic energy consists of the translational kinetic energy and the rotational energy (Dorren and Berger 2005; Chau et al. 2002; JRA 1983):

$$E_k = E_t + E_r = E_t(1 + \beta) \tag{2}$$

in which E_t is the translational kinetic energy, E_r is the rotational energy, and β is the ratio of the rotational energy to the translational kinetic energy, which is recommended to be taken as 0.1 as suggested by JRA (1983).

The translational kinetic energy E_t is expressed as:

$$E_t = 0.5MV^2 \tag{3}$$

in which M is the rock mass (kg) and V is the rock translational velocity (m/s).

The rotational energy E_r is expressed as:

$$E_r = 0.5Iw^2 \tag{4}$$

in which I is the moment of inertia for the rock (kg m^2) and w is the is the angular velocity (rad/s). For a sphere, $I = 0.4Mr^2$, where r is the radius (m).

The JRA (Japan Road Association) (1983) reported that the ratios (β) of the rotational energy to the translational kinetic energy are below the 10% line for about half of the field data, and all field data are below the 40% line. Chau et al. (2002) indicated that the ratio (β) increases with the slope angle γ which the rock moving on, and achieves a maximum value of 0.4 at about $\gamma = 40^\circ$. Physically, the 40% line can be interpreted as the upper boundary for the case of spherical boulder impact (Chau et al. 2002).

For energy-absorbing layers that are used as protection for structures subjected to rockfall, the penetration distance l may be obtained by measuring it. For the structures themselves and the rockfall blocks, the penetration distance are difficult to measure because they are extremely short and determined by the interaction between the rock and the object. Kawahara and Muro (2006), Pichler et al. (2005, 2006), Montani-Stoffel (1998), Volkwein et al. (2011), Zhang et al. (1995), Hungr et al. (1984), and Timoshenko and Goodier (1970) determined the penetration distance from the modulus of elasticity and the Poisson ratio of the rock and the object.

The dimensional homogeneity can be used to describe the impact force with the help of the Buckingham theorem (Buckingham 1915). First, seven important factors have to be specified: F = impact force from the rock block on the object (N), E_k = the kinetic energy of the rock block (J), α = the angle between the movement direction of the block and the surface of the object (degree), E_1 = the modulus of elasticity of the rock material (Pa), E_2 = the modulus of elasticity of the object (Pa), μ_1 = the Poisson ratio of the rock material, and μ_2 = the Poisson ratio of the object. The angle α can be expressed as *sin* α (dimensionless) instead of the parameter α.

Zhang et al. (1995) obtained a calculation model for the maximum impact force. The relationship between the modulus of elasticity of rock and object is derived from the elastic contact theory by Hertz. A mixed modulus of elasticity was introduced by them as shown in Eq. 5:

$$E = \frac{E_1E_2}{E_2(1-{\mu_1}^2) + E_1(1-{\mu_2}^2)} \tag{5}$$

in which E is the mixed modulus of elasticity (Pa).

Most Poisson ratios are in a range of 0 to 0.3. Therefore, the role of the Poisson ratio is limited in Eq. 5: the values of $(1-{\mu_1}^2)$ and $(1-{\mu_2}^2)$ in Eq. 5 are in the range of 0.91 to 1 if the Poisson ratio is smaller than 0.3. To simplify the analysis, the items $(1-{\mu_1}^2)$ and $(1-{\mu_2}^2)$ are set to a value 1. In Eq. 5, the influences of the moduli of elasticity of rock material and object are equal but their values may be greatly different. A coefficient is introduced to correct for these differences. Then, a new formula for the mixed modulus of elasticity is obtained:

$$E = \frac{E_1E_2}{E_1 + kE_2} \tag{6}$$

in which k = the coefficient of the influence of the modulus of elasticity of the rock and the object. The influence of the modulus of elasticity of the rock material is the same as the influence of the modulus of the object when $k = 1$. The coefficient k is determined by experiments of impact force with different moduli of elasticity of the rock and the object.

The moduli of elasticity of rock are in the order of magnitude of 50 GPa. The moduli of elasticity of most of the stiff structure are in the same order of magnitude. In this case, the moduli of elasticity of the rock and the object are both important if the coefficient k is in the order of 1 in Eq. 6. But for the cushion layers or the not stiffness of the structure which are important in the interaction with the rockfall, the values of the moduli of elasticity may be very low. The elastic modulus of rock E_1 can be ignored in that case and only the elastic modulus of the cushion layer or the not stiffness of the structure E_2 plays an important role in the interaction with the rockfall if the coefficient k is in the order of 1 in Eq. 6.

The object will be removed by a rockfall if the mass or the resistance of the structure is too small. In this case, the modulus of elasticity is not important during an interaction. In this study, the mass or resistance of the structure are considered to be too strong to be removed. In this case, the modulus of elasticity is important during the interaction.

Only four parameters (F, E_k, E, sin α) are defined as follows in terms of their three primary dimensions length $\{L\}$, mass $\{M\}$, and time $\{T\}$:

$$\begin{aligned}\{F\} = \{ML/T^2\}, \{E_k\} = \{ML^2/T^2\}, \{E\} \\ = \{M/L/T^2\}, \{\sin\alpha\} = \{0\}\end{aligned} \tag{7}$$

According to Buckingham (1915), one can then obtain two dimensional parameters. Two dimensionless Π groups were set up by combining the dimensional parameters (F, E_k, E) and the dimensionless parameter sinα.

The Buckingham theorem (Buckingham 1915) provides a method for computing sets of dimensionless parameters from given variables, even if the form of the equation remains unknown. Buckingham's theorem provides a way to generate sets of dimensionless parameters. The solutions for the exponents of the Π groups (Buckingham 1915) are:

$$\Pi_1 = \frac{F}{\left(EE_k^2\right)^{1/3}} \quad (8)$$
$$\Pi_2 = \sin\alpha$$

In the Π groups, one dimensionless parameter (for example, $F/(EE_k^2)^{1/3}$ in this study) is the function of the other dimensionless parameters (for example, sinα in this study). So the impact force F can be expressed as:

$$F = \left(EE_k^2\right)^{1/3} f(\sin\alpha) \quad (9)$$

$f(\sin\alpha)$ is the function of sinα. The impact interaction is in a particular situation when the angle between the movement direction of the rock block and the surface of the object is 90°. We set $f(\sin\alpha) = c$ when $a = 90^o$. Then, the impact force F can be expressed as:

$$F = c\left(EE_k^2\right)^{1/3} \quad (10)$$

in which c is a coefficient to be determined. The physical meaning of the coefficient c in Eq. 10 is a constant which is the dimensionless ratio of $F/(EE_k^2)^{1/3}$ when $a = 90^o$. One can obtain the simultaneous equation of the impact force F from the Eq. 2 and Eq. 10:

$$F = (1+\beta)^{2/3} c\left(EE_t^2\right)^{1/3} \quad (11)$$

If the rotational energy $E_r = 0$ ($\beta = 0$), the energy E_k can be given instead by the translational kinetic energy E_t in Eq. 10. If the ratio $\beta = 0.1$, the parameter $(1 + \beta)^{2/3}$ is 1.07. If the ratio $\beta = 0.4$, the parameter $(1 + \beta)^{2/3}$ is 1.25.

The exponents of the modulus of elasticity E, and the kinetic energy of rock E_k (for the mass of rock M, and the velocity of rock V) in the models of impact force used in this study and other studies are listed in Table 1. The range of the exponents of the mass of the rock block M is 0.6–1 in other studies against 0.67 in this study. The range of the exponent of the velocity of rock V is 1–2 in other studies against 1.33 in this study. The range of the exponent of the modulus of elasticity E is 0.4–0.5 in other studies against 0.33 in this study. The range of the exponent of the kinetic energy of rock E_k is 0.6–1 in other studies against 0.67 in this study. This shows that the model of Eq. 10 is close to other models but that the most important difference is the dimensional homogeneity in Eq. 10.

Experimental setup and testing procedure

The experimental setup is illustrated in Fig. 1. A flume is used with a length of 10 m, width of 0.38 m, and a height of 0.5 m. An inner flume which is 8 m long, 0.2 m wide, and 0.2 m high is put in the center of the flume to assure that the impact area is always in the center area of the impact board. The balls were released from the upper part of the inner flume, and they rolled to strike the impact board at the lower end of the flume. The slope angle of the flume was 3- 12°. The impact area is located between the four transducers as shown in Fig. 1b. The impact board was connected to the fixed board by all four transducers. The fixed board can be adjusted to vary the angle α between the movement direction of the ball and the surface of the impact board from 90 to 5°. The maximum impact forces were obtained by adding the maximum forces measured at the four transducers (see Fig. 2). Two sets of force transducer with different force measurement ranges were used in the experiments: (1) 50–500 N and (2) 500–5000 N. So the impact force range that can be measured in the experiments is 200–20,000 N.

The time interval of every sample with impact force is 0.0002 s. There is always an initial value of impact force in each transducer. The maximum values of initial force for each transducer are between −20 and +20 N. So the maximum values of initial force for the 4 transducers together are between −80 and +80 N. But the maximum values of initial force in the experiments are between −40 and +40 N, and most of them are between −20 and +20 N. In the experiments, the maximum measured impact forces are between 225 and 15,583 N. The measurement errors during the experiments may thus be in the range of 5 to 18% when the maximum impact force is less

Table 1 Comparison of the exponents of mass, velocity, modulus, and energy between the models in the different studies

References	Mass M (kg)	Velocity V (m/s)	Elastic modulus E (GPa)	Energy E_k (J)
This study	0.67	1.33	0.33	0.67
Zhang et al. (1995)	0.6	1.2	0.4	0.6
Labiouse et al. (1996)	0.6	1.2	0.4	0.6
Kawahara and Muro (2006)	0.67	1.2	0.4	–
Pichler et al. (2005)	1	2	–	1
CREEG (1978)	1	1	0.5	–
Montani-Stoffel (1998)	0.6	1.2	0.4	0.6
Calvetti et al. (2005)	–	–	–	0.67
Berthet-Rambaud et al. (2004), and Delhomme et al. (2007)	1	–	–	1
Ronco et al. (2009)	–	–	0.4	0.6

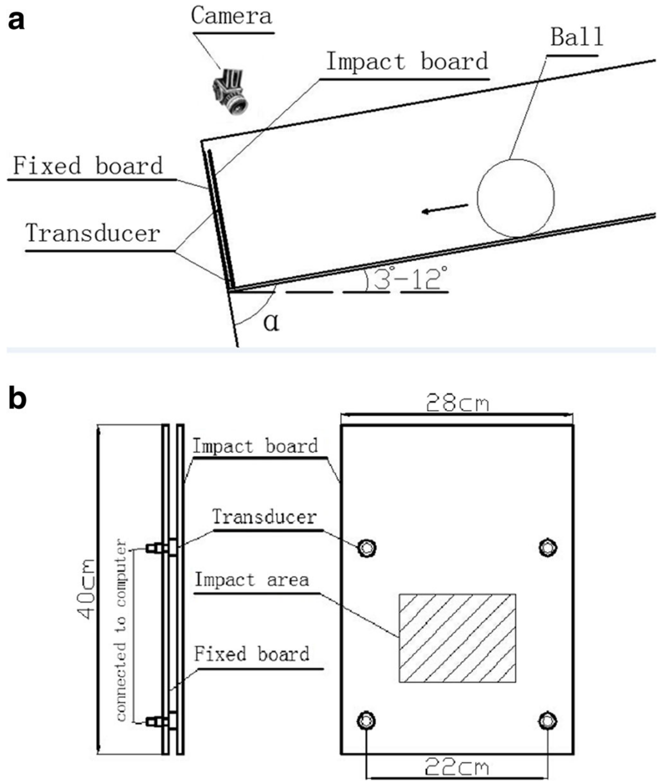

Fig. 1 Experimental setup

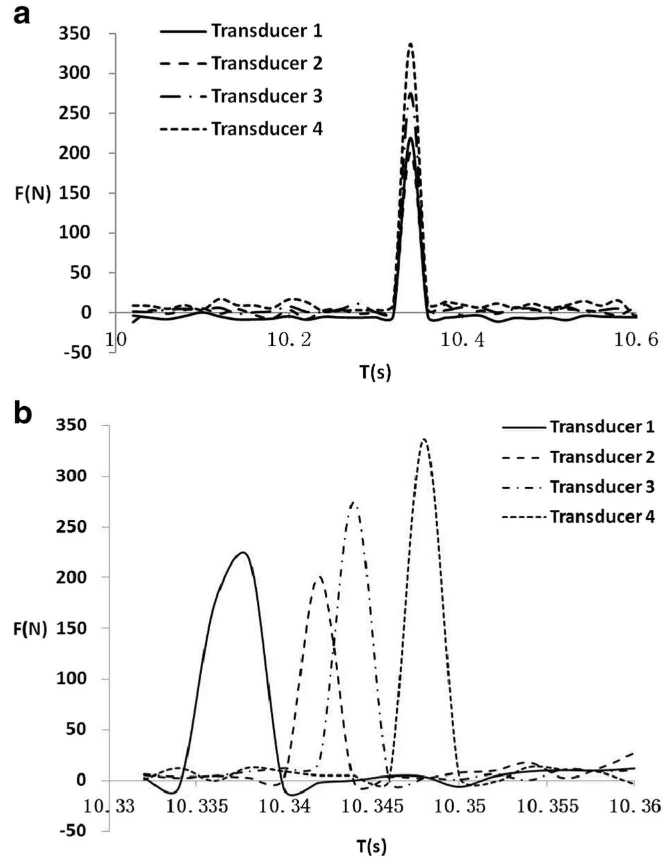

Fig. 2 The maximum impact force measured in four transducers. **a** The time between 10.0 and 10.6 s. **b** The time between 10.33 and 10.36 s

than 800 N. But for most of our experiments, the errors are less than 5% because the maximum impact forces are larger than 800 N, and the expected total initial forces are between −20 and +20 N.

Figure 2 shows the times at which the maximum impact forces were measured by the four transducers in one typical test. The times of maximum impact forces in 4 transducers are between 10.338 and 10.348. There is a 0.01-s time difference for these maximum forces because of the error of the instruments. But there is only one peak of impact force for each transducer during the collision; the time difference can be ignored.

The velocity of the balls before the collisions could be calculated with help of a video camera which provided 25 records per second. From the displacement of the ball between the last two records before the impact, the impact velocity was calculated. The relationship of the ratio β and the velocity at impact was measured before the tests. The ratio β decreases with increasing impact velocity. The ratio β was in the range of 0.365–0.40 in all tests. With the relationship of the ratio β, the mass of the ball, and the impact velocity, one can determine the kinetic energy of the ball. The mass of the balls of different materials is listed in Table 2. Five different ball materials were used in the experiments: iron, granite, marble, sandstone, and wood. Three types of marble ball were used in experiments. Three different materials were used for the impact board: steel, concrete, and wood. The thickness of these boards is 0.2, 5, and 3.5 cm, respectively. The material of the fixing board was steel with 0.4 cm thickness. The modulus of elasticity of the granite ball, marble ball, and sandstone ball were measured before the tests. The modulus of elasticity of the iron ball, and the wooden ball, as well as the steel, concrete, and wooden impact board, were obtained from the Mechanical Engineering Design Manual (MEDM 2008) (See Table 2).

Experiments with different impact kinetic energies, moduli of elasticity of the balls, moduli of elasticity of the impact boards, and different impact angles between the movement direction of the ball and the surface of the impact board were carried out: in total, 124 experimental runs.

Test results

Maximum impact force and kinetic energy

The relationship between the impact kinetic energy E_k and the maximum impact force F in Eq. 10, was investigated first for marble balls with the masses $M = 1.56$, 3.56, and 11.86 kg, respectively, and an impact board of concrete. Figure 3 shows the relationship between the impact force F and kinetic energy E_k. The exponent value 2/3 fits excellently to the experimental data, so the relationship between the maximum impact force F and the kinetic energy E_k in Eq. 10 is consistent with the experiments. This relationship can be used to analyze the relationship between the maximum impact force F and the modulus of elasticity E.

Maximum impact force and modulus of elasticity

To determine the relationship between the modulus of elasticity E and the maximum impact force F in Eq. 10, experiments were carried out with iron, granite, marble, sandstone, and wood balls,

Table 2 Parameters of the materials used in the experiments

	Material	Mass (kg)	Diameter (cm)	Elastic modulus (GPa)
Ball	Iron	5.02	10.7	155
Ball	Granite	1.64	10.2	83.34
Ball	Marble	1.56	10.3	47.38
Ball	Marble	3.56	13.6	47.38
Ball	Marble	11.86	20.3	47.38
Ball	Sandstone	1.26	19.8	26.45
Ball	Wood	1.32	18.5	11
Impact board	Steel	–	–	175
Impact board	Concrete	–	–	19.66
Impact board	Wood	–	–	11

respectively, impacting on steel, concrete, and wooden impact boards. When the coefficient $k = 2$, we obtain the best fit line in Fig. 4, which shows the relationship between $F/E_k^{2/3}$ and the modulus of elasticity E. The exponent value 1/3 fits excellently to the experimental data of $F/E_k^{2/3}$ and the modulus of elasticity E, which shows that the relationship between the maximum impact force F and the modulus of elasticity E in Eq. 10 is consistent with the results of the experiments. The coefficient c is obtained from the best fit line in Fig. 4: $c = 0.35$. The relationships between the impact force F, the kinetic energy E_k, and the modulus of elasticity E can be used to determine the relationship between the maximum impact force F and the angle α of impact between the movement direction of the ball and the surface of the object.

Maximum impact force and impact angle

A marble ball with the mass $M = 1.56$ kg and a steel impact board were used in the experiments to determine the relationship between the maximum impact force and the impact angle α. The impact angle was set at 5, 10, 20, 30, 60, and 90°, respectively. Figure 5 shows the relationship between $\sin\alpha$ and F/F_o, in which $F_o = E^{1/3}E_k^{2/3}$. The ordinate in Fig. 5 is the ratio between the measured maximum impact force F and the calculated maximum impact force F_o. The line in Fig. 5 with the exponent value 0.5 fits excellently with the F/F_o and $\sin\alpha$ values. From the relationship between the maximum impact force F, the kinetic energy E_k, the modulus of elasticity E, and the angle α, one can experimentally determine the coefficient c in Eq. 10.

Maximum impact force model

To obtain the coefficient c in Eq. 10, experiments with large maximum impact force F were carried out. An iron ball with the mass $M = 5.02$ kg was used for these experiments, and a steel impact board. The largest maximum impact force measured in these experiments was 15,583 N. Figure 6 shows the relationship between the measured maximum impact force F and the calculated maximum impact force $(EE_k^2)^{1/3}\sin^{0.5}\alpha$ in Eq. 10. The coefficient c is obtained from the best fit line in Fig. 6 and it is the same as the fit line in Fig. 4: $c = 0.35$. Figure 6 shows that the relationship between the maximum impact force F and the modulus of elasticity E in Eq. 10 is consistent with the experiments. Figure 6 indicates as well that the relationship between the maximum impact force F and the kinetic energy E_k, the modulus of elasticity E, and the

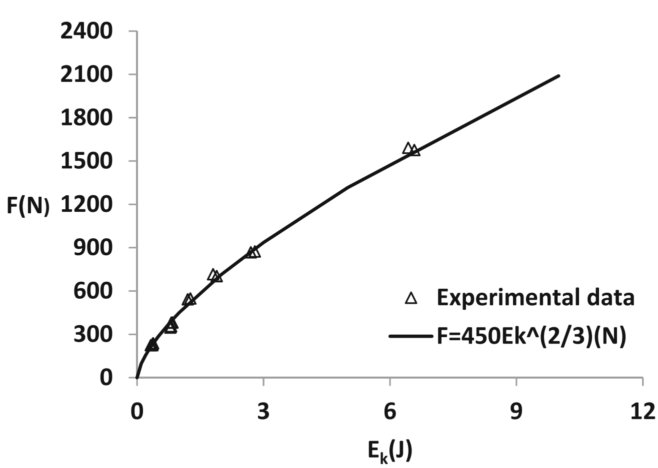

Fig. 3 Relationship between force F and kinetic energy E_k. Tests with marble balls with a mass $M = 1.56$, 3.56, and 11.86 kg, respectively, and a concrete impact board

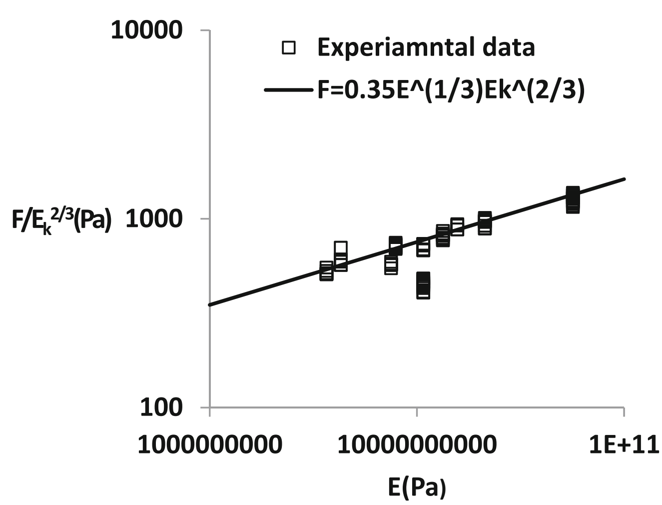

Fig. 4 Relationship between impact force F and modulus of elasticity E

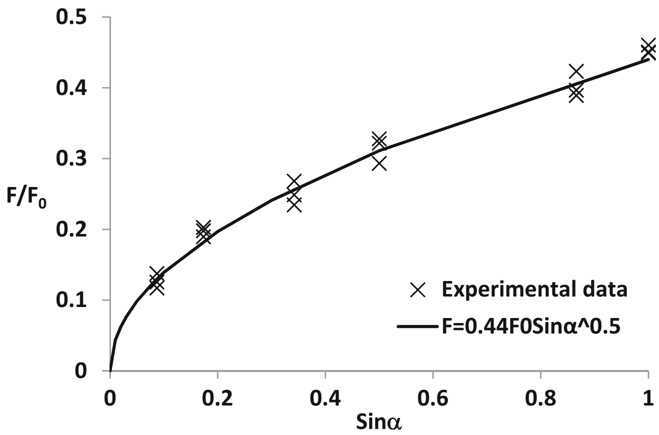

Fig. 5 Relationship between impact force F/F_0 and impact angle α. F_0 is the calculated impact force from the relationship in Fig.4 at right angle

angle α in Eq. 10 is consistent with the experiments. So the maximum impact force F can be expressed as (Eq. 12 and Eq. 13):

$$F = 0.35(EE_k{}^2)^{1/3}(\sin\alpha)^{0.5} \quad (12)$$

Where

$$E = \frac{E_1E_2}{E_1 + 2E_2} \quad (13)$$

In Eq. 13, $E \approx E_2$ if $E_1 \gg E_2$. In the case of rock fall into a cushion layer, the elastic modulus of the rock E_1 is far larger than the elastic modulus of the cushion layer E_2 (see Table 1 and Table 3), so the elastic modulus of rock E_1 can be ignored and only the elastic modulus of the cushion layer E_2 plays an important role in the impact force. In this case, the role of the elastic modulus E in Eq. 12 and Eq. 13 is similar to the role of the elastic modulus of the cushion layer in the model of Kawahara and Muro (2006). And our model can be validated with the experimental results of the impact force with the cushion layer.

Table 3 Elastic modulus of cushion layers of different materials

Soil	Elastic modulus (MPa)
Very soft clay	0.3–0.35
Soft clay	2–5
Medium clay	4–8
Stiff clay	7–18
Sandy clay	30–40
Silt	7–20
Loose sand	10–25
Dense sand	50–80
Dense sand and gravel	100–200

Validation

Validation of the impact angle

Yuan et al. (2014) conducted some experiments with an iron ball and a steel board. The ratio β was in the range of 0.33–0.36 in their experiments. They compared the different maximum impact forces for different impact angles α: 30, 60, and 90° for balls with the same mass and velocity. Figure 7 shows the relationship between $sin^{0.5}\alpha$ and F/F_1. The ordinate in Fig. 7 is the ratio between the measured maximum impact force F with the angle α = 30 or 60° and the measured maximum impact force F_1 with the impact angle α = 90°. Figure 7 shows that the experimental data fall around the fit line. This validates the relationship between the maximum impact force and the impact angle in our Eq. 12.

Validation of tests at different scales

Yang and Guan (1996) conducted some experiments with iron hammers falling on a soft clay layer with hammer masses M = 0.51, and 0.81 kg. The falling heights of the hammer were in the range of 0.05–0.3 m. The velocities of impact were calculated from the falling height of the hammer. The ratio β = 0 in their experiments. The thickness of the soft clay layer varied from 1.5 to 5 cm. The impact force decreased with increasing thickness of the soft clay layer. The impact force did not decrease more when the

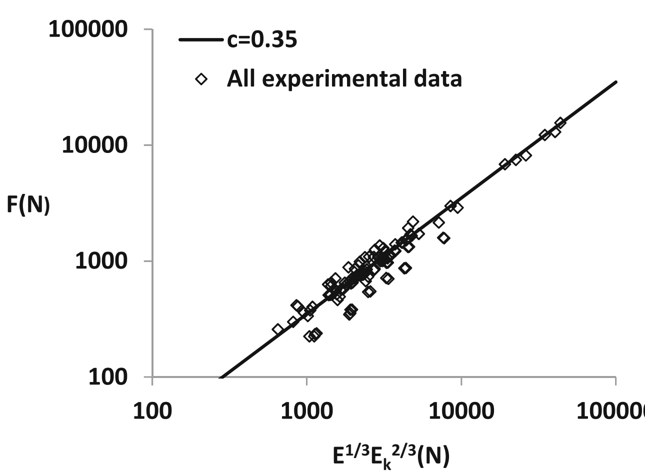

Fig. 6 The coefficient c of our impact force model

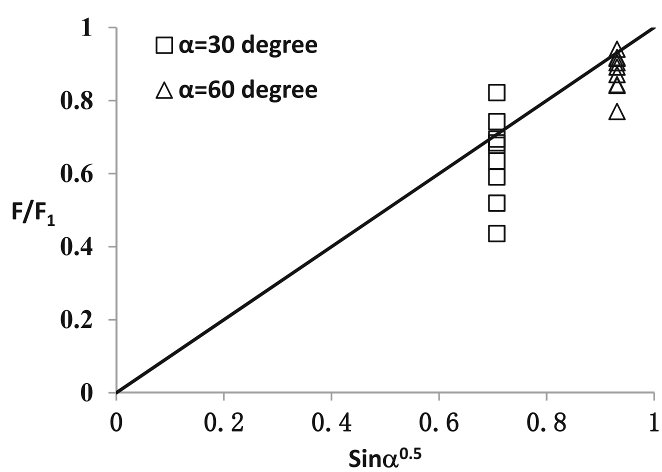

Fig. 7 Validation of impact angle α by the experiments of Yuan et al. (2014)

clay layer thickness was reaching 5 cm. The experimental data with the thickness of 5 cm were used in the validation because the thickness of the clay layer was assumed to be sufficient to absorb energy. The resulting impact forces were in the range of 21.4 to 119 N. The modulus of elasticity of cushion layers of different materials is listed in Table 3 (http://wenku.baidu.com/view/21c43d2a4431b90d6d85c71f.html, in Chinese). The modulus of elasticity of the soft clay layer E_2 is taken as the mean value of the range for soft clay in Table 3: 3.5 MPa. Figure 8 shows the relationship between the measured maximum impact force F_m and the calculated maximum impact force F_c.

Labiouse et al. (1996) conducted some experiments with falling rock blocks on a reinforced concrete slab covered by fill material. The maximum impact forces were caused with rock blocks masses M = 500, and 1000 kg, the falling heights H varied from 0.25 to 9.5 m, and the thickness of the gravel cushion was 0.5 m (Labiouse et al. 1996). The ratio β is 0 in their experiments. Figure 8 shows that the maximum impact force F_c calculated with Eq. 12 with a value of 3200 kPa for the modulus of elasticity (Labiouse et al. 1996) is far less than the measured maximum impact force F_m by Labiouse et al. (1996). But if a mean value of 150 MPa (in Table 3) of a mixture of dense sand and gravel is used instead of the value of 3200 kPa, the calculated maximum impact force F_c from Eq. 12 is close to the measured maximum impact force F_m by Labiouse et al. (1996) (see Fig. 8).

Pichler et al. (2005) conducted some experiments with granite rock blocks falling into a trench filled with 60% of well-graded gravel, and 40% of angular stones. This cushion layer was compacted to a density of 1800 kg/m^3. The masses of the rock blocks used were 10,160 and 18,260 kg. The impact velocities were in the range of 6.26 to 19.23 m/s, and the maximum impact forces were in the range of 1.53 to 7.95 MN (by their estimation for g = 0 in their calculation, Pichler et al. 2005). The ratio β is 0 in their experiments. The test layer was 2 m thick and the maximum penetration depth was 0.85 m. In Table 3, the modulus of elasticity of soils increases with increasing grain size. The largest grain size in Table 3 is sand and gravel. The soil used in Pichler et al. (2005) was dense gravel and stone. Extrapolating the values for the modulus of elasticity of dense sand, and dense sand and gravel, the modulus of elasticity of a mixture of dense gravel and stone may be in the range of 200–500 MPa with a mean value of 350 MPa. A plot of the estimated maximum impact force F_m of the prototype test of Pichler et al. (2005) and the calculated maximum impact force F_c from Eq. 12 is shown in Fig. 8.

The maximum impact forces F_m measured in the experiments of Yuan et al. (2014) are plotted in Fig. 8 against the maximum impact forces F_c calculated from Eq. 12. Figure 8 shows that the measured maximum impact force F_m and calculated maximum impact force F_c of Yang and Guan (1996) are also in very good agreement. The measured maximum impact forces F_m in the experiments of Yuan et al. (2014) and Labiouse et al. (1996) are a factor 1000 larger than the maximum impact force of Yang and Guan (1996), but they show the same trend, as is also the case for the data from the very large scale tests of Pichler et al. (2005). There are five orders of magnitude between the prototype sizes of the tests producing these four data sets. The maximum impact forces in the tests of Pichler et al. (2005) are of the same order of magnitude as rock falls into cushion layers in the field, and we assume them to be very close to the order of magnitude of rock fall impacting on the houses, roads, and bridges. So the maximum impact force model in this study is suitable for the calculation of maximum impact forces in a very large range, and may be also suitable for the calculation of maximum impact forces for prototype scales because of its dimensional homogeneity.

We compared our results with some earlier papers on the maximum impact force with or without cushion layer. Kawahara and Muro (2006) introduced a method to calculate the maximum impact force on a cushion layer caused by a rockfall in Japan (JRA 1983):

$$F = 21.08(Mg)^{2/3}\lambda^{2/5}H^{3/5} \tag{14}$$

in which g is the acceleration of gravity (9.81 m/s^2), λ is the Lame's constant of a sand cushion (kPa), and H is the drop height (m).

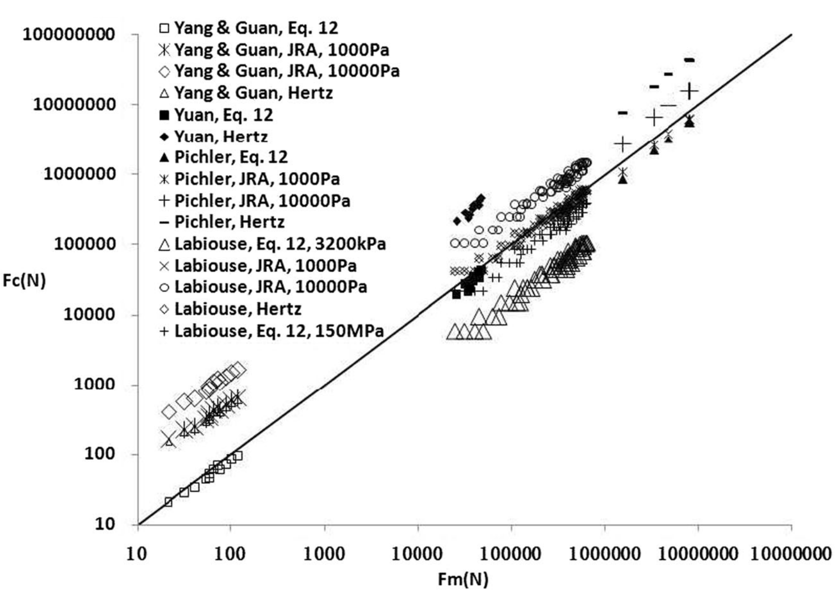

Fig. 8 Comparison of the tests and the calculations at different scales by other authors

The general value of λ used is 1000 kPa, but the actual values of λ range from 1000 to 10,000 kPa due to variation of the density of the sand cushion. The above equation is derived from an elastic contact theory by Hertz, assuming that the rockfall block is a rigid sphere having a specific gravity of 2.65 and the sand cushion has a plane and horizontal surface (Kawahara and Muro 2006). This formulation is not a dimensionally homogeneous formulation.

Labiouse et al. (1996) obtained an equation for the maximum impact force on a cushion layer also derived from Hertz's elastic contact theory and modified by their experimental results. Yuan et al. (2014) used Hertz's elastic contact theory to calculate the maximum impact force with and without cushion layer. These approaches can be expressed as Hertz's formulation of maximum impact force:

$$F = 1.16\left(\frac{E_1E_2}{E_1(1-{\mu_2}^2)+E_2(1-{\mu_1}^2)}\right)^{2/5} M^{3/5}R^{1/5}V^{6/5} \quad (15)$$

in which R is the radius of the falling block (m). This is a formulation with dimensional homogeneity.

Figure 8 shows the comparison of the experimental results on cushion layer by Pichler et al. (2005), Labiouse et al. (1996), and Yang and Guan (1996), and the calculation results of JRA's formulation with λ = 1000 kPa, JRA's formulation with λ = 10,000 kPa, and Hertz's formulation (the items (1-μ_1^2) and (1-μ_2^2) are set to 1 because the μ_1 and μ_2 are unavailable). The calculations by the Hertz's formulation are larger than the experiments of Pichler et al. (2005), and the calculations by the JRA's formulation with λ = 1000 kPa are close to the experiments. The calculations by the Hertz's formulation, the JRA's formulation with λ = 1000 kPa are close to results of the experiments of Labiouse et al. (1996). The calculations by the JRA's formulation with λ = 1000 kPa and λ = 10,000 kPa, the Hertz's formulation are larger than the experiments of Yang and Guan (1996). Figure 8 shows the comparison of the experimental results without cushion layer by Yuan et al. (2014) and the calculation results of the Hertz's formulation. The calculations by the Hertz's formulation are larger than the experiments of Yuan et al. (2014).

Discussion

We used the Buckingham theorem (Buckingham 1915) to obtain dimensional homogeneity for the impact force. The theory and the laboratory experiments show that the results are reasonably good: the data calculated in this study are close to the experimental data of other authors in a very large range of scales. But there is an uncertainty error in the comparison of the calculation model with the test data of Pichler et al. (2005). The masses, velocities, and impact angles in the calculation of the impact force of tests of Pichler et al. (2005) are exactly known, but the modulus of elasticity of the rock blocks and the cushion layer were not determined. The modulus of elasticity of the rock E_1 is not important as we have shown in the Test Results chapter that the modulus of elasticity E is very close to the modulus of elasticity of the cushion layer E_2. The cushion layer consisted of 60% of wide range-grained gravel, and 40% of edged stones, and was compacted. The modulus of elasticity was estimated from Table 3. The error of Eq. 12 is estimated by comparing the data of Pichler et al. (2005) and the data calculated with Eq. 12 for different values of the modulus of elasticity. If the modulus of elasticity is taken as 350 MPa in Fig. 8, the error is about 30–34% for 4 tests, and is 45% for 1 test; but if the modulus of elasticity is assumed to be 700 MPa, the error will be less than 17% for 4 tests, and will be 31% for 1 test. But it may not be possible to achieve a modulus of the cushion layers higher than 150 MPa, so the modulus of elasticity of 350 MPa and especially that of 700 MPa may be too high. The suggestion to improve the calculations by using a different modulus of elasticity, however, is still based on an estimated modulus value, so the result is still uncertain. On the other hand, the estimated values of Pichler et al. (2005) may also cause errors: the maximum impact force was calculated by a model of impact kinematics which was deduced from experimental acceleration measurements. The results of our model as compared to the results of Pichler et al. show that both models are reasonably good. In the tests of Yang and Guan (1996), and Labiouse et al. (1996), the modulus of elasticity of the cushion layer E_2 is very important for the calculation of the maximum impact force, and the values of modulus of elasticity of the cushion layer were obtained by experiments. The modulus of elasticity of 150 MPa used for the calculation of Labiouse's data by our model is an estimation, which is not validated by experimental results. However, the modulus of elasticity of the cushion layer is needed to obtain the key parameter for the impact force model in the future. Large-scale experiments at field scale may be a good choice to validate our model in the future.

The modulus of elasticity of the cushion layer determines the results of the maximum impact force calculated by Eq. 12 and the Hertz's formulation. The maximum impact forces calculated by Eq. 12 are far less than the experimental values when the modulus of elasticity of the cushion layer is 3200 kPa as provided by Labiouse et al. (1996), but the values calculated by Eq. 12 are close to the experimental values when the modulus of elasticity of the cushion layer is taken as 150 MPa from Table 3. We consider the Labiouse's value of 3200 kPa to be unrealistic. This value corresponds to soft clay materials which usually are not used for this purpose. On the other hand, the comparisons of the maximum impact forces calculated by Hertz's formulation will be good when smaller values for the moduli of elasticity of the cushion layers are chosen for the calculating of the data as presented by Yang and Guan (1996) (3.5 kPa) and Pichler et al. (2005) (5.5 MPa). So the Hertz's formulation and Eq. 12 may have different coefficient values due to differences in the value of the modulus of elasticity of the cushion layer. But the calculations by Hertz's formulation are larger than the values of experiments without the cushion layer of Yuan et al. (2014). Our model presented in this paper may be more suitable for cases with and without cushion layer.

The modulus of elasticity of 350 and 700 MPa may be too high, and the modulus of elasticity of 3200 kPa may be too low for the materials of cushion layer. For the materials of cushion layer constructed from sand, gravel, and coarse gravel and cobbles, the most common moduli of elasticity are from 40 to 100 MPa. Figure 9 shows the comparison of the tests and the calculations of Eq. 12 and Hertz's formulation at 40 and 100 MPa by Pichler's tests and Labiouse's tests. For the calculations of Eq. 12, the average errors are 68.2% (40 MPa, minus) and 56.8% (100 MPa, minus) for Pichler's tests, and 62.6% (40 MPa, minus) and 49.3% (100 MPa, minus) for Labiouse's tests. For the calculations of Hertz's formulation, the average errors are 126.0% (40 MPa,

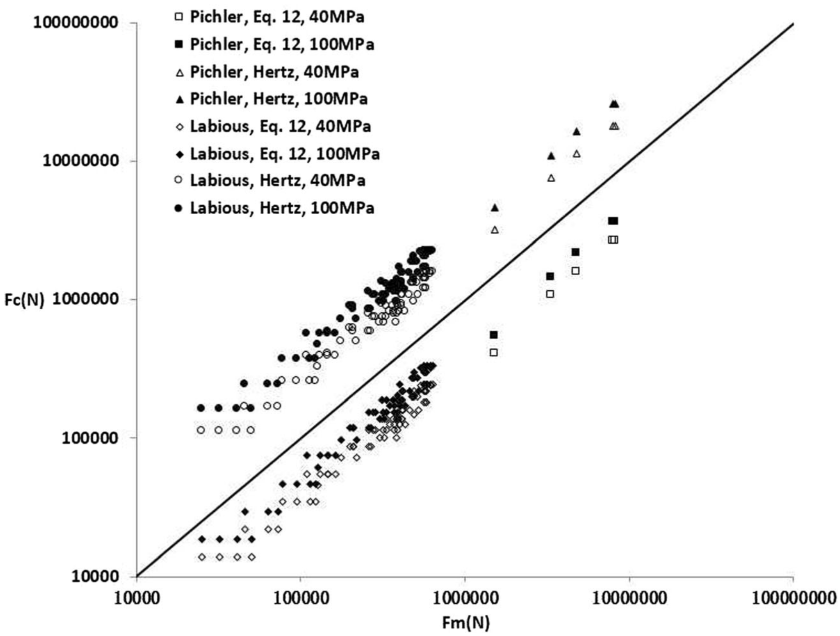

Fig. 9 Comparison of the tests and the calculations of Eq. 12 and Hertz's formulation at 40 and 100 MP by Pichler's tests and Labiouse's tests

positive), and 225.9% (100 MPa, positive) for Pichler's tests, and 157.2% (40 MPa, positive), and 271.0% (100 MPa, positive) for Labiouse's tests. Figure 9 shows both the Hertz's formulation and Eq. 12 have moderate errors in the value of the common modulus of elasticity of the cushion layer. More research works should be conducted to revise Eq. 12 to fit the calculations of the common modulus of elasticity of the cushion layer in the future.

The calculations of the maximum impact forces by the JRA's formulation are very good when compared with the results of the large-scale experiments of Labiouse et al. (1996) and Pichler et al. (2005). However, the calculated values for the small-scale experiments of Yang and Guan (1996) are too large. The large error of JRA's formulation may be the result of the fact that the formulation is not dimensionally homogeneous. The JRA's formulation (Eq. 14) cannot be used for the case without cushion layer.

Pichler et al. (2005) found that the maximum impact force is a function of the penetration depth, the impact duration, the height of the fall (or the velocity of rock), and the mass of the rock boulders. They determined the penetration depth and the impact duration experimentally. In our study, no penetration was found and no impact duration was measured or estimated. We used instead the modulus of elasticity of the ball and the modulus of elasticity of the impacted object. This made the model of this study more simple and direct. For the rock blocks and some object materials, the modulus of elasticity can easily be obtained. For the cushion layers, which are widely used in protection measures, the modulus of elasticity can vary over a wide range of soil materials: from loose to dense, from fine to coarse, and from soft to stiff. It is difficult to determine an exact value for the modulus of elasticity which is important for the impact force model. The penetration depth and the impact duration as were used in the model of Pichler et al. may be considered in the modulus of elasticity of the cushion layer. A redefined modulus of elasticity of the cushion layer including the effects of penetration depth and impact duration should be investigated in future research.

Indentation of the impacting object was observed in the tests of Yang and Guan (1996), Labiouse et al. (1996), and Pichler et al. (2005). No indentation of the impacted object was considered in the model of this paper. The measured maximum impact force F_m of Yang and Guan (1996) and Pichler et al. (2005), and the calculated maximum impact forces F_c based on our model are in very good agreement. Our model may also be used for the calculation of maximum impact force with a cushion layer because the modulus of elasticity of the cushion layer is considered in the model. This hypothesis may be right when the thickness of the cushion layer is sufficient, as in the tests of Yang and Guan (1996) and Pichler et al. (2005). Labiouse et al. (1996) already pointed out that the coefficient of the maximum impact force equation depends on the thickness and the material of the cushion. When the cushion layer is thin, our model is not suitable for the calculation of impact force because the cushion layer cannot absorb enough energy. The required thickness for absorbing enough energy increases with an increase of the maximum impact force. This is why the largest value of thickness in Yang and Guan's experiment was chosen for our validation. The relationship between the maximum impact force and the necessary thickness of the cushion layer should be researched in the future.

The thicknesses of the steel, concrete, and wood impact boards are 0.2, 5, and 3.5 cm, respectively. They were fixed on a steel board with a thickness of 0.4 cm. Although no distortions were observed during the experiments, the types of boards and the stiffness of the force measuring system may cause errors in the results of the experiments. Higher stiffness of the force measuring system and thicker impact boards and fixing board may be introduced in the future work to obtain more reliable data.

For impact between rigid bodies, the mass and geometry (width) of the object might play a significant role, as it affects the displacement of object and rock block and the duration of the impact, which are two crucial factors for the calculation of the maximum impact force. In this study, the mass and geometry of the object are considered to be too large to be moved. This is a limitation of the proposed methodology when the displacement of object comes to the impact between two rigid bodies. The validation of the results of the large-scale tests of Pichler et al. (2005) shows that our impact model may be applied at in situ scales because of the dimensional homogeneity. However, the experiments described in this paper have relatively small maximum impact forces with only minor deformations. Stronger impacts may cause more plastic deformation and the results may diverge from the linear relationships in the model. More research work should be conducted at larger in situ scales in the future to validate our model.

A simplification was made in the Buckingham theorem to obtain dimensional homogeneity for the impact force. This simplification made it easy to obtain the coefficients by experiments. The factors (1-μ_1^2) and (1-μ_2^2) were given a value 1 (the Poisson ratio is set to 0) in Eq. 3 because most Poisson ratios are in a range of 0–0.3. But the Poisson ratio may be up to 0.5 for some types of material. This may cause considerable errors in the calculation of the impact force. To evaluate the role of the Poisson ratio in the model of the impact force, more experiments should be conducted with materials with different Poisson ratios, especially for materials with a high value of Poisson ratio.

Conclusion

An experimental study of maximum impact force of rock blocks was carried out based on the dimensional analysis. The results show that the maximum impact force of a rock block onto an object is a function of the kinetic energy of the block, the angle of impact between the movement direction of the block and the surface of the object, and the modulus of elasticity of the rock and object materials. The maximum impact force can also be expressed as a function of the ratio β, the translational kinetic energy of the block, the angle of impact, and the modulus of elasticity of rock and object materials. The validation with test results from other authors over a large range of scales shows that our model works reasonably well, and may be used to calculate the maximum impact force at the full field scale.

In our view, our maximum impact force model offers a new and exciting way to calculate the maximum impact force not only on an object, but also onto a cushion layer. However, to improve the understanding of the impact force and the properties of the cushion layer, future research is needed to provide realistic values for the modulus of elasticity of the cushion layer, to discover the relationship between the penetration depth and the impact duration with the modulus of elasticity of the cushion layer, to determine the relationship between the maximum impact force and the required thickness of a cushion layer, and to determine the role of the Poisson ratio of the materials of the block and the object.

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